Quantum Mechanics A PHY-319 Problems 3

Issue: Tuesday 24th January 2012 **Hand-in:** 16:00 Thursday 2nd February 2012
Hand-In questions **1** and **2**
Please post your solutions in the box outside the 1st floor departmental offices

QUESTION 1.

Last week we modelled the hexatriene molecule as an infinite square well; this time we will model it as a finite square well 0.6 nm wide and 14 eV deep.

a) Calculate ζ_0 for this potential. Hence calculate the number of bound states (energy levels) in this well.

b) A graphical solution yields values for η… Given that these are: $\eta_1 = 1.34$, $\eta_2 = 2.66$, $\eta_3 = 3.95$ and $\eta_4 = 5.17$ for the first four levels, calculate the corresponding energies in eV

c) Suppose that once again the molecule is exposed to electromagnetic radiation of just the right wavelength to excite an electron from the n = 3 to the n = 4 level. Calculate this wavelength.

d) By considering the interval over which η_n is found, prove that in the limit $\zeta_0 \to \infty$ we recover the infinite square well energy solutions.

QUESTION 2.

An electron is confined to a 8Å wide, 8eV deep, one-dimensional potential well.

a) Calculate the number of electronic bound states for such a well.

b) By using a spreadsheet (or other numerical method) evaluate all allowed η_n values for such a situation to three significant figures, hence calculate the allowed energies. Please print out any graphs or other computer output so that we can check your method!

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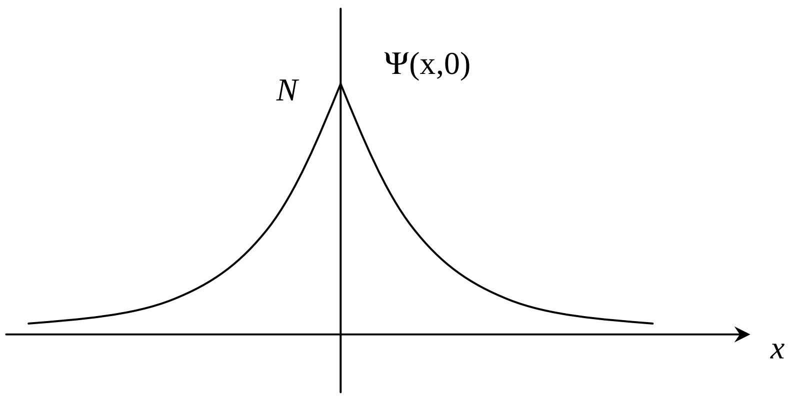
Exercise Class Questions.

1. At $t = 0$ a particle in one dimension has the wave function shown below.



This can be written as: $\Psi(x,0) = Ne^{-a|x|}$

or as:

$\Psi(x,0) = Ne^{-ax}$ for $x \geq 0$

and

$\Psi(x,0) = Ne^{ax}$ for $x \leq 0$

a) By normalising the wave function, $\Psi(x,0)$, determine *N*.

b) Evaluate $\langle x \rangle$ and $\langle x^2 \rangle$, and hence the uncertainty in position, Δx .

c) What is the probability of finding the particle in the range $-\Delta x \leq x \leq +\Delta x$?

2. Consider a particle confined to a one dimensional finite quantum well, depth V_0 and width L.

a) Sketch the potential for such a system.

b) Write down the Schrödinger equation for the classically allowed and classically forbidden regions.

c) Obtain solutions for the classically allowed and classically forbidden regions.

d) Sketch the first <u>three</u> wavefunctions and probability densities for this potential.

useful "stuff":

$$\int xe^{Cx}dx = \frac{e^{Cx}(Cx-1)}{C^2} \qquad \text{and} \qquad \int x^2 e^{Cx}dx = \frac{e^{Cx}(2-2Cx+C^2x^2)}{C^3}$$

m_e = 9.11x10^{-31}kg = 0.511MeV/c^2 $\hbar$ = 1.05x10^{-34}Js = 6.58x10^{-16}eVs

c = 3x10^{8}ms^{-1}