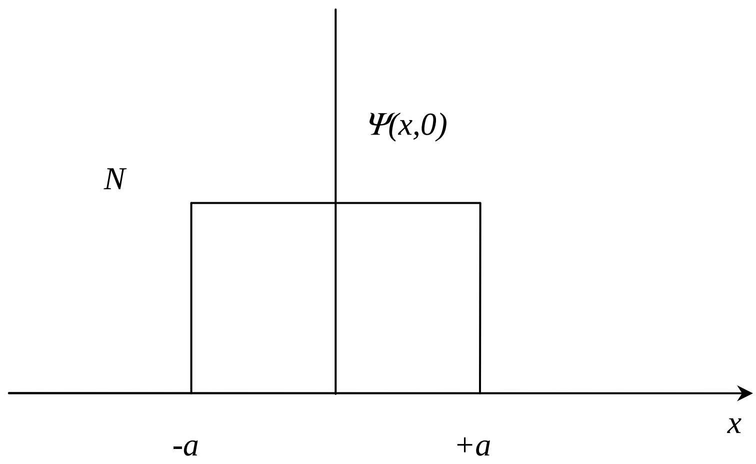
Quantum Mechanics A   PHY-319   Problems 1

Issue: Tuesday 10th January 2012   **Hand-in:** 16:00 Thursday 19th January 2012
Hand-In questions **1**
Please post your solutions in the box outside the 1st floor departmental offices.

QUESTION 1.
At $t = 0$ a particle in one dimension has the wave function shown below:



This can be written as:
$\Psi(x,0) = N$ for $-a \le x \le +a$
and
$\Psi(x,0) = 0$ elsewhere

a) There is something not quite right with this wavefunction. What is it?
b) By normalising the wave function, $\Psi(x,0)$, determine *N*.
c) Evaluate $\langle x \rangle$ and $\langle x^2 \rangle$, and hence the uncertainty in position, Δx.

Exercise Class Questions.

1) Revision exercises in the use of complex numbers:

Write down the complex conjugate, Ψ^*, and the mod-squared, $|\Psi|^2$, of each of the following wave functions (all symbols except i are real unless otherwise stated):

a) $\Psi(x,t) = e^{-i\pi x} e^{-it}$

b) $\Psi(x,t) = e^{-ax^2} e^{-\frac{iEt}{\hbar}}$

2) Normalisation of a wavefunction:

a) The ground state wavefunction for the simple harmonic oscillator can be written as:

$$\Psi(x,t) = \psi(x) e^{\frac{-iE_0 t}{\hbar}} \text{ with } \psi(x) \text{ given by } \psi(x) = N e^{-\frac{ax^2}{2}}$$

By normalising the wave function, $\Psi(x,t)$, determine N.

b) The ground state wavefunction for for a particle in an infinite potential well, between *0* and *L*, can be written as:

$$\Psi(x,t) = N \sin\left(\frac{\pi}{L} x\right) e^{\frac{-iE_1 t}{\hbar}} \text{ for } 0 \le x \le L$$

$$= 0 \qquad \text{elsewhere}$$

By normalising the wave function, $\Psi(x,t)$, determine N.

Some useful maths: $\int_{-\infty}^{\infty} e^{-Cx^2} dx = \sqrt{\frac{\pi}{C}}$ and $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$