Quantum Mechanics PHYS2B22
Problem Sheet 1

Work in to my pigeon-hole by Monday January 24th

1) In Compton's original scattering experiment, X-rays from a particular atomic source (the so-called Mo $K\alpha$ line) having a well-defined initial wavelength $\lambda = 0.711\mathring{A}$ were scattered from a material. Using Compton's formula, predict the final wavelength of the scattered X-rays for deflection angles of (i) 0^o, (ii) 90^o, (iii) 135^o, and (iv) 180^o. [**4**]

2) Consider a diffraction experiment like that of Davisson and Germer. For normal incidence ($\theta_i = 90^o$) and reflection angle θ_r (as defined in lectures), constructive interference gives rise to a peak in the diffraction pattern when $n\lambda = a\cos\theta_r$, where n is an integer (the diffraction order), λ is the de Broglie wavelength and a is the spacing between atomic planes.

a) Consider the case of electron diffraction. If the electrons (charge $-e$, mass m_e) are first accelerated by a potential Φ, show that their de Broglie wavelength is $\lambda = h/\sqrt{2e\Phi m_e}$. Hence calculate λ for $\Phi = 54$V. [**4**]

b) In the experiment, a peak (not necessarily with $n = 1$) is seen for 54eV electrons at normal incidence when $\theta_r = 40^o$. At what value of θ_r would a peak of the same order (same value of n) be seen for 75eV electrons? Is there a peak for 25eV electrons? [**4**]

c) What energy neutrons would also produce a peak (at the same order) at $\theta_r = 40^o$ (Neutron mass $= 1.67 \times 10^{-27}$Kg). [**4**]

3a) At some instant of time, a particle moving in one-dimension is described by the un-normalized wavefunction $\Psi(x) = e^{-\mu|x|}$. Find the value of the normalization integral $N = \int_{\infty}^{\infty} dx\, |\Psi(x)|^2$ and hence construct a properly normalized wavefunction from Ψ. [**5**]
Is $\Psi(x)$ a physically allowed wavefunction? State your reasons. [**2**]

3b) What is the probability that the particle will be found in the region $0 \leq x \leq 2$? [**5**]

4) Pellets of mass m, initially at rest, are dropped a distance H onto the floor. The uncertainty principle implies that the pellets cannot be assumed to fall straight down from rest. Allow for uncertainty in the initial position (Δx_0) and momentum (Δp_0) in a horizontal direction to show that the spread of pellets on the floor can be estimated to be

$$\Delta x \simeq \Delta x_0 + \frac{1}{m}\Delta p_0 \sqrt{2H/g},$$

where g is the acceleration due to gravity. **[5]**
Hence find a formula for the *minimum* spread on the floor [*Hint: use the uncertainty principle*]. **[5]**
Calculate the minimum spread for pellets of mass 1Kg dropped from a height of 1m (use $g = 10\text{ms}^{-2}$). **[2]**

5) **[For tutorial discussion – no marks.]** We normally expect physical properties of a system to be unchanged if we make an arbitrary change in the origin of the potential energy scale (i.e. add an arbitrary constant to the potential energy). What effect would this change have on a) the time-independent Schrödinger equation, b) the time-dependent Schrödinger equation, and c) the time-dependence of the wavefunction? Would any of these changes have observable consequences?