POSTULATES OF QUANTUM THEORY: SUMMARY OF MAIN EQUATIONS

1) In quantum theory the value of an observable which we can measure (e.g. energy, momentum, angular momentum, etc.) is obtained from an EIGENVALUE EQUATION – see Eq. 4.1

$$\hat{a}\phi_n = \lambda_n \phi_n$$ where $\hat{a}$ is an operator (LHO) and,

ϕ_n is an Eigenfunction and
λ_n is an Eigenvalue.

2) ORTHOGONALITY of the ϕ_n (Eq. 4.5)

$$\int \phi_n^* \phi_m d\tau = 0$$ unless m = n

NB. If the ϕ_n are ORTHONORMAL the Eigenfunctions are both orthogonal and normalised. So for the case where n = m:

$$\int \phi_n^* \phi_n d\tau = 1$$

3) COMPLETENESS of the ϕ_n (Eq. 4.6) : Any well-behaved Wavefunction can be expanded as a superposition

$$\Psi(x) = \sum C_n \phi_n$$

Then the probability of measuring the Eigenvalue λ_n corresponding to the Eigenfunction, ϕ_n is (Eq. 4.7)

$$C_n^* C_n$$

For orthonormal Eigenfunctions, ϕ_n (Eq. 4.8)

$$\sum_n C_n^* C_n = 1$$

Also, by orthonormality,

$$C_n = \int \phi_n^*(x) \psi(x) dx$$

Once a measurement has yielded $\lambda = \lambda_n$ all subsequent measurements of the system will yield the same value since $\Psi(x) = \sum C_n \phi_n$ reduces to $\Psi(x) = \phi_n$.

4) THE EXPECTATION VALUE of an Operator is the average obtained after many measurements of an observable of a large number of identical systems. It is defined as (Eq. 4.12)

$$\langle \hat{a} \rangle = \int \Psi^* \hat{a} \Psi d\tau$$

But if we know Ψ in terms of its Eigenfunctions, we do not have to calculate the integral explicitly, since we know (Eq. 4.13)

$$\langle \hat{a} \rangle = \sum_n \lambda_n |C_n|^2$$

A word about notation:

$$\langle \varphi | \hat{H} | \varphi \rangle = \int \varphi^* \hat{H} \varphi \, d\tau$$

Where φ^* is the complex conjugate of the Wavefunction, φ and $\hat{H}$ is an Operator, e.g. the Hamiltonian for the energy.