# ELECTRIC AND MAGNETIC FIELDS

## ASSIGNMENT 7

**Questions 1-4 count for 95% of the marks and question 5 for 5%**

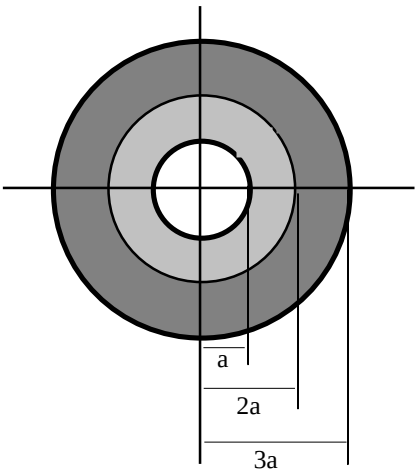
**Q1** A parallel plate capacitor has square plates of dimensions 5 x 5 cm and separation 2 mm. The space between the plates is filled with a material of dielectric constant $\kappa = 4$. The charges on the plates are $\pm 5 \times 10^{-8}$ C. Find the following:

(i) the capacitance;
(ii) the potential difference between the plates;
(iii) the magnitude of the electric field between the plates;
(iv) the electric energy density between the plates; and
(v) the total electric energy.

**Q2** A slab of material with dielectric constant of 2 is partly inserted a distance x between the plates of a parallel plate capacitor of plate separation d and sides a and b.

Show that the capacitance is $C = \dfrac{\varepsilon_0 b(a + x)}{d}$.

**Q3** A spherical capacitor consists of an inner conducting sphere of radius a and a thin outer conducting spherical shell of radius 3a. The space between the conductors is filled with two spherical layers ofdifferent dielectrics:




Radius $a \rightarrow 2a$ : dielectric constant K
Radius $2a \rightarrow 3a$ : dielectric constant 2K

The inner sphere has charge +Q and the outer shell has charge -Q.

(a) Draw a diagram of the capacitor showing the electric field lines.

(b) Write down the expressions for the magnitude of the electric field as a function of radius, r, for (c)

(i) $r < a$ (inside the inner sphere);
(ii) $a < r < 2a$ (inside the first dielectric layer);
(iii) $2a < r < 3a$ (inside the second dielectric layer); and
(iv) $r > 3a$ (outside the whole thing).

**Note:** No proof is required here, just a few words of explanation in each case.

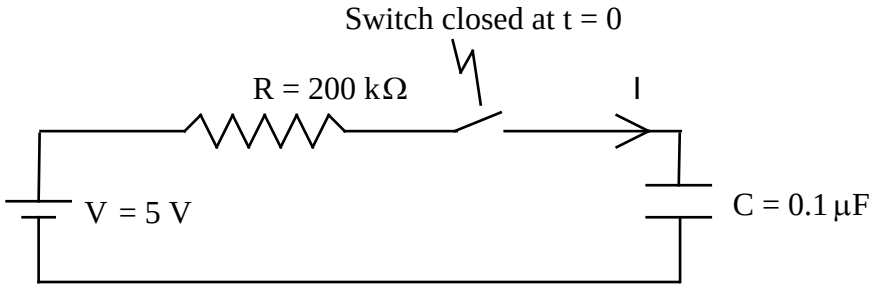
(c) Integrate the electric field between the inner and outer conductors to show that the magnitude of the potential difference between them is

$$\Delta V = \frac{7Q}{48\pi K\varepsilon_0 a}.$$ Which conductor is at a higher potential?

**Hint:** You will need to split the line integral into two parts, corresponding to the two different dielectrics.

(d) What is the capacitance?

**Q4** An RC circuit is connected in series with a battery of voltage V,




(i) Write down equations for the time-varying voltage across the capacitor, V(t), and the current, I(t).

(ii) At what time is the charge on the capacitor equal to 90% of its final value?

(iii) Derive an expression for the energy stored in the capacitor as a function of time. At what time does the energy stored by the capacitor reach 90% of its final value?

**Q5** In the lectures, we integrated the energy density of the electric field over all space to derive this equation for the electric energy, U, of a sphere of radius R, containing total charge Q distributed uniformly inside it:

$$U = \frac{3Q^2}{20\pi\varepsilon_0 R}.$$

Show that the same formula can be derived by finding the total amount of work which must be done to assemble the sphere of charge.

**Hint:** Consider an intermediate stage at which a sphere of radius r (< R) has already been assembled. Find the work done in adding a thin shell to it, and integrate the resulting expression.You may find it easier to work in terms of the charge density, ρ - it will cancel out in the end.