ELECTRIC AND MAGNETIC FIELDS

ASSIGNMENT 5

Questions 1 - 3 count for 95% of the marks and Question 4 for 5%.

Q1: The atmospheric electric field is uniform with a magnitude of 100 V m^{-1} between the ionosphere and the ground. Assume that the ionosphere and the ground are both perfect conductors, and that the ionosphere is at a height of 120 km.

(i) Find the electric energy density of the atmosphere in J m^{-3}.

(ii) Assume that the radius of the Earth is 6400 km. What is the total electric energy of the atmosphere?

Q2: Four equal point charges, Q, are arranged in a square of side a. Show that the total electric energy of the system is

$$U_{tot} = \frac{Q^2}{4\pi\varepsilon_0 a}\left[4+\sqrt{2}\right].$$

Hints: • Draw a simple sketch • Use the high degree of symmetry to simplify the problem.

Q3: A solid conducting sphere, of radius a, is at the centre of a thin conducting shell of radius b (where b > a). The inner sphere has a total charge -Q, and the shell has a total charge +Q.

(i) Draw a diagram showing the electric field lines (remember the spheres are conductors).

(ii) Show that the electric energy density at a point in the space between the sphere and the shell is given by

$$u(r) = \frac{Q^2}{32\pi^2\varepsilon_0 r^4}$$ where r is the distance of the point from the centre.

(iii) Integrate this expression over the volume of the space between the sphere and the shell to show that the total electric energy of the system is

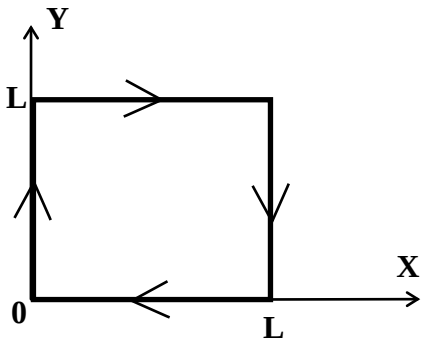
$$U_{tot} = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{b-a}{ab}\right].$$

(iv) Show that for the case of b >> a this reduces to the result derived in the lectures for an isolated spherical condictor with total charge Q.

Q4: Show that the electric field

$\overline{\mathbf{E}}(x,y) = (ay)\,\hat{\mathbf{i}} + (bx)\,\hat{\mathbf{j}}$,

where a and b are constants, is conservative only if a = b .



How to do it:

Integrate $\overline{\mathbf{E}} \cdot \mathbf{d}\overline{\mathbf{L}}$ around the square loop shown.