ELECTRIC AND MAGNETIC FIELDS

ANSWERS TO WEEK 4 ASSIGNMENT

30

Q1: (a) Position 1: (-28.81, 0, 0) pm Position 2: (11.71, 11.71, 11.71) pm

Using the formula for the potential at distance r from a point charge, the values of the potential at each of the two positions due to the α-particle at (0,0,0) are

12

~ 6 for method
~ 2 each for the three correct answers

$$V_1 = \frac{2e}{4\pi\varepsilon_0 r_1} \qquad \text{where } r_1 = [(28.81^2 + (0)^2 + (0)^2]^{1/2} = 28.81 \text{ pm}$$

$$V_2 = \frac{2e}{4\pi\varepsilon_0 r_2} \qquad \text{where } r_2 = [(11.71)^2 + (11.71)^2 + (11.71)^2]^{1/2} = 20.28 \text{ pm}$$

$e = 1.602 \times 10^{-19}$ C $\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2\, N^{-1}\, m^{-2}$

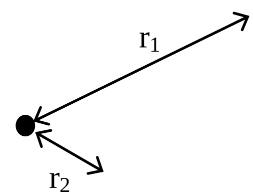
So **$V_1 = 100$ V $V_2 = 142$ V $\Rightarrow$ $V_2 - V_1 = 42$ V**

(b) $V_2 > V_1 \Rightarrow$ we would need to do work on a positive charge to go from 1 to 2, but a negative charge would be **pulled** along by the field. Therefore the electron **loses** potential energy in moving to position 2.

10 for a very clear answer

Mark on method + convincing demonstration of an understanding of work and PE

Or: r_2 is less than r_1, so electron would be pulled in towards the new position, so losing potential energy.



(c) PE lost is ΔU = (Charge)(Potential difference) = (e)(ΔV)

ΔU = 42 eV or $\Delta U = (42)(1.602 \times 10^{-19}) = 6.73 \times 10^{-18}$ J.

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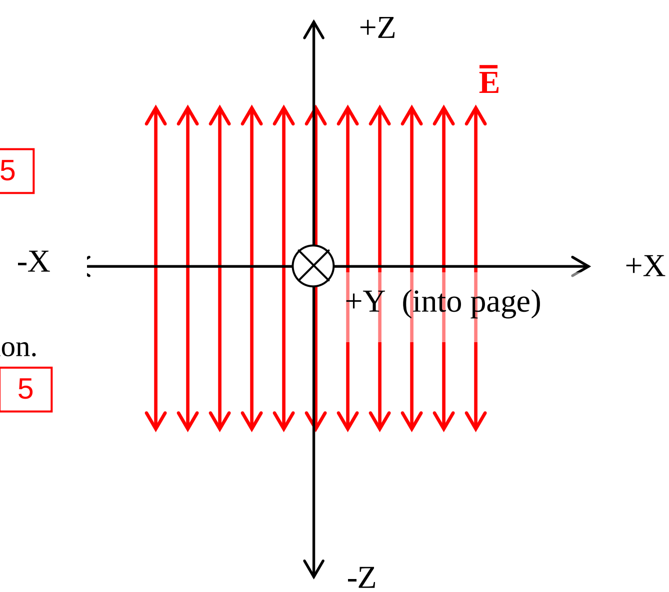
Q2 Infinite plane of surface charge density 8 μC m^{-2} is in the x-y plane.



(a) $\overline{\mathbf{E}}$ is everwhere perpendicular to the plane of charge. So for positive values of z, $\overline{\mathbf{E}}$ is in the +z direction. 5

(b) For negative values of z, $\overline{\mathbf{E}}$ is in the -Z direction. 5

(c) a: (0,0,2) cm b: (0,0,5) cm

From the lecture notes, $E = \sigma/(2\varepsilon_o)$ and $V(z) = -Ez$

$\sigma = 8 \times 10^{-6}$ C m^{-2} and $\varepsilon_o = 8.85 \times 10^{-12}$ F m^{-1} $\Rightarrow$ $E = 4.52 \times 10^{5}$ N C^{-1} for **both** points (since E is independent of z) 4 + 4

$V_1 = -4.52 \times 10^5(2 \times 10^{-2}) = -9.04 \times 10^3$ V $= -9.04$ kV 4

$V_2 = -4.52 \times 10^5(5 \times 10^{-2}) = -2.26 \times 10^4$ V $= -22.6$ kV 4

(a) -2 mC is moved from a to b.

Work done is $|W| = |q(\Delta V)| = |(-2 \times 10^{-3})(22.60 - 9.04) \times 10^3| = 27.1$ Joules 4

(Note that b is further away from the plane than a, so a negative charge must be **pushed** along the electric field direction from a to b – i.e., work must be done on the charge.) 2

(b) -2 mC charge moved from (0,0,5) to (5,5,5).

Work done = zero as change in z is zero. 3

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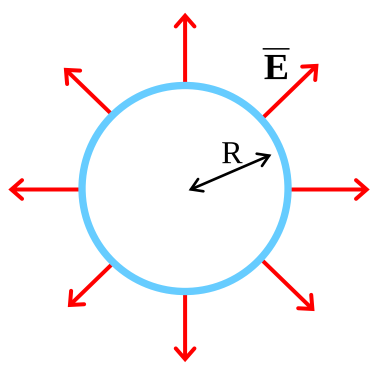
Q3: (a) Charge is uniformly distributed over the surface

$\Rightarrow$ spherical symmetry

$\Rightarrow$ field pattern is as for a point charge outside the sphere.



10 for diagram + explanation of the field pattern and expressions for E inside and outside

Diagram is essential here

Deduct ~ 3 if E is given as non-zero inside.

For r > R: $E = 0$

Inside the sphere, E must be zero, because a spherical Gaussian surface of radius < R encloses no charge.

For r > R: $$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

where Q = total charge on surface = $4\pi R^2\sigma$ $\Rightarrow$ $$E = \frac{R^2\sigma}{\varepsilon_0 r^2}.$$

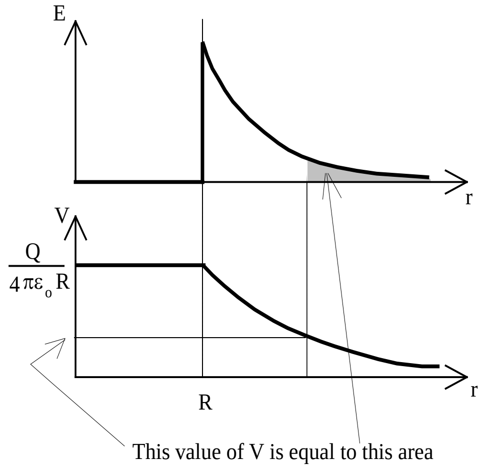
10 for using correct relationship between the electric field and V. Working out the magnitude and determining the sign afterwards.is the recommended

Relationship between potential and electric field: $$|\Delta V| = \left|\int \overline{\mathbf{E}} \cdot d\overline{\mathbf{L}}\right|$$

Taking V = 0 at r = ∞, to find V at radius r we integrate along a radial path from ∞ to r:
(i) For r > R, we have:

$$|V(r)| = \left|\int_{\infty}^{r} \overline{\mathbf{E}} \cdot d\overline{\mathbf{L}}\right| = \left|\int_{\infty}^{r} \overline{\mathbf{E}} \cdot d\overline{\mathbf{r}}\right| = \left|\int_{\infty}^{r} E dr\right| = \left|\frac{R^2\sigma}{\varepsilon_0}\int_{\infty}^{r} \frac{1}{r^2} dr\right|$$

$$\Rightarrow \quad |V(r)| = \left|\frac{R^2\sigma}{\varepsilon_0 r}\right|.$$



We would need to **push** positive charge from ∞ to r against the field, so V(r) is **positive**.

At the surface of the shell,

$$V(R) = \frac{R\sigma}{\varepsilon_0}$$

(ii) For r < R, we do not need to do any further work to move the charge in, because E = 0

$$\Rightarrow \quad \overline{\mathbf{E}} \cdot d\overline{\mathbf{L}} = 0.$$

Therefore, V(r) is constant, with the same value as that at the surface.

8 for a good sketch

(b) At surface of shell, PE of point charge q is (q)(V at surface) $\Rightarrow \quad U_R = \frac{qR\sigma}{\varepsilon_0}$

At radius 2R, the PE is $U_{2R} = \frac{qR^2\sigma}{\varepsilon_o(2R)} = \frac{qR\sigma}{2\varepsilon_o} \quad \Rightarrow \quad \Delta U = \frac{qR\sigma}{2\varepsilon_o}.$

This amount of PE has been converted to KE

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{qR\sigma}{2\varepsilon_o} \quad \Rightarrow \quad v = \left[\frac{qR\sigma}{\varepsilon_o m}\right]^{1/2}$$

7:

4 for method

and

3 for correct answer