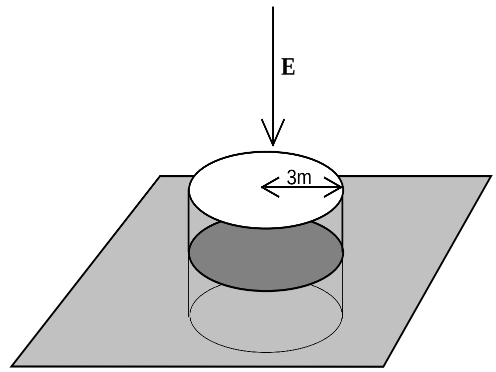
ELECTRIC AND MAGNETIC FIELDS

ASSIGNMENT 3

Note: Questions 1 - 3 count for 95% of the marks, and question 4 for 5%.

Q1 The Earth's electric field points vertically downwards with $E = 80\ N\ C^{-1}$. A non-conducting cylindrical box with radius 4m is half buried in the ground (and so half filled with soil) as shown. Regard the soil as a perfect conductor.



(i) What is the total electric flux through the box?

(ii) What is the total electric charge enclosed by the box? Where is this charge located?

(iii) What are the answers to (i) and (ii) for a spherical box of the same radius, half buried in the ground?

Q2 A long plastic rod has radius a and carries electric charge uniformly distributed within it. The charge contained in a 1-metre length is λ.

(a) Draw side-view and axial-view diagrams showing the rod and the electric field pattern.

(b) Use Gauss's law to find the magnitude of the electric field for two regions (i) $r < a$, and (ii) $r > a$.

(c) Find the total electric flux through a co-axial cylindrical surface of radius a/2 and length 5 m.

Q3 For a sphere of radius R containing uniform charge density (charge per unit volume) of ρ, the magnitude of the electric field, E, at a distance r from the centre is given by

$$E = \frac{\rho r}{3\varepsilon_0} \quad \text{for } r < R \qquad E = \frac{\rho R^3}{3\varepsilon_0 r^2} \quad \text{for } r > R \quad \text{(derived in lectures).}$$

(i) Sketch the variation of E with radial distance.
(ii) For a particular sphere of charge, of radius R, the following measurements are made:

$r = 10$ mm, $E = 3.77 \times 10^5\ N\ C^{-1}$ $\qquad$ $r = 40$ mm, $E = 1.88 \times 10^5\ N\ C^{-1}$.

Determine: (a) The charge density, ρ $\qquad$ (b) The radius, R $\qquad$ (c) The total charge, Q

Hint: There are three possibilities: both points inside the sphere; one inside and one outside; or both outside. A thorough answer should explain why only one of these possibilities is consistent with the information given.

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Q4 The electric charge of the proton is modelled as an exponentially varying charge density

$$\rho = \frac{e}{8\pi b^3} \mathrm{Exp}\left[-\frac{r}{b}\right] \quad \mathrm{C\,m^{-3}} \qquad \text{where } b = 2.3 \times 10^{-16}\ \text{m.}$$

(i) Use Gauss's law to show that the magnitude of the electric field as a function of radial distance is given by

$$E_r = \frac{e}{8\pi\varepsilon_0 r^2}\left[2 - \frac{r^2}{b^2}\mathrm{Exp}\left(-\frac{r}{b}\right) - 2\left(\frac{r}{b}+1\right)\mathrm{Exp}\left(-\frac{r}{b}\right)\right].$$

(ii) Calculate E at r = 7x 10^{-16} m. How does this compare to the field due to a point charge at the central position?

Note: You will need to use the following standard integral:

$$\int x^2 e^{-x} = -x^2 e^{-x} - 2e^{-x}(x+1)$$

Hint: Apply the step-by-step procedure for using Gauss's law. The only non-trivial bit is determining how much charge is inside the Gaussian surface.