ELECTRIC AND MAGNETIC FIELDS
ANSWERS TO WEEK 2 ASSIGNMENT

20 **Q1 (a)** Coulomb's Law gives the magnitude of the field due to a point charge. The direction of the field is radially outwards from the nucleus or proton.

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$Q = e = 1.60 \times 10^{-19}$ C
$r = 5.29 \times 10^{-11}$ m
$\varepsilon_o = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

$\Rightarrow$ $E = 5.17 \times 10^{11}\ N\,C^{-1}$ 8

(b) The magnitude of the force required to make a particle of mass m move in a circle of radius r with speed v is

6 for using this $F = \frac{mv^2}{r}$.

This force is provided by the electrostatic attraction exerted by the proton on the electron.

Therefore, letting m_e be the mass of the electron, we have

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r} \quad \Rightarrow \quad v = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}}$$

6 for working out v correctly

$e = 1.60 \times 10^{-19}$ C, $r = 5.29 \times 10^{-11}$ m
$\varepsilon_o = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$, $m_e = 9.109 \times 10^{-31}$ kg

$\Rightarrow$ $v = 2.19 \times 10^6\ m\ s^1$

25 **Q2 (a)** The person must have a negative charge to generate an upward electric force opposing gravity. Let m be the mass of the person and -Q be the charge.

Magnitude of electric force $F_e = QE$
Magnitude of gravitational force $F_g = mg$
If these are equal, then $Q = mg/E$

$E = 150\ N\ C^{-1}$ $m = 50$ kg $g = 9.81\ m\ s^{-2}$ $\Rightarrow$ $Q = -3.27$ Coulombs

10
3 for sign
7 for magnitude
(at least 4 for using QE = mg)

(b) For two 50-kg people, 500 m apart, with charges of -3.27 C, the repulsive force is

$$F = \frac{Q^2}{4\pi\varepsilon_o r^2} = \frac{3.27^2}{4\pi\varepsilon_0 (500)^2} = \mathbf{3.85 \times 10^5\ N}$$

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Acceleration: $a = \frac{F}{m} = \frac{3.85 \times 10^5}{50} = \mathbf{7692\ m\ s^2}$

Acceleration due to gravity $g = 9.81\ m\ s^2$, so **a = 784 g** 8

(c) The charge required to oppose gravity is so large that enormous electrostatic repulsive forces would be generated. For instance, in the case above, the two people would be able to float, but their horizontal acceleration would be almost 800g - far higher than the maximum survivable of about about 50g.

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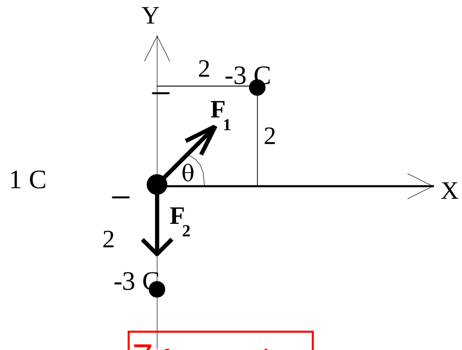
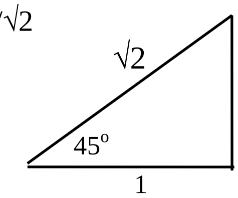
25 **Q3** Diagram showing the charges and the forces on the 1 C charge:



7 for good diagram(s)

$\theta = 45^o \Rightarrow \cos\theta = \sin\theta = 1/\sqrt{2}$



Expressing the two forces $\overline{\mathbf{F}}_1$ and $\overline{\mathbf{F}}_2$ in terms of their X and Y components, we get

$$\overline{\mathbf{F}}_1 = \frac{(1)(3)}{4\pi\varepsilon_0(2\sqrt{2})^2}\left[\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}\right]$$

So $$\overline{\mathbf{F}}_1 = \frac{3}{32\pi\varepsilon_0}\left[\frac{1}{\sqrt{2}}\hat{\mathbf{i}} + \frac{1}{\sqrt{2}}\hat{\mathbf{j}}\right].$$

12 for the correct method: expressing the forces in terms of their orthogonal components and adding them as vectors.

$$\overline{\mathbf{F}}_2 = \frac{(1)(3)}{4\pi\varepsilon_0 2^2}\left[0\,\hat{\mathbf{i}} - \hat{\mathbf{j}}\right] = \frac{3}{16\pi\varepsilon_0}\left[0\,\hat{\mathbf{i}} - \hat{\mathbf{j}}\right].$$

The resultant force is therefore $$\overline{\mathbf{F}}_1 + \overline{\mathbf{F}}_2 = \frac{3}{32\pi\varepsilon_0}\left[\frac{1}{\sqrt{2}}\hat{\mathbf{i}} - (2 - \frac{1}{\sqrt{2}})\hat{\mathbf{j}}\right]$$

or $$\overline{\mathbf{F}}_1 + \overline{\mathbf{F}}_2 = (2.4 \text{ x } 10^9)\hat{\mathbf{i}} - (4.4 \text{ x } 10^9)\hat{\mathbf{j}}.$$

6 for the correct final answer

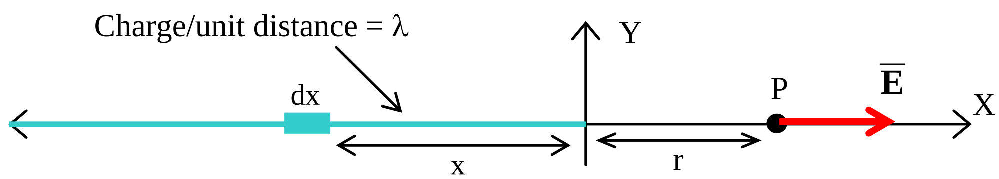
25 **Q4** **(a)** To apply the Principle of Superposition to this problem:

1. We divide up the rod into many very short elements, dx.
2. We regard an element dx as a point charge, and use Coulombs Law to find its contribution to $\overline{\mathbf{E}}$ at point P.
3. To find the total field at P, we integrate over the whole length of the rod

8 for a good account of the principle of superposition with some relevance to the problem.

(b) Clearly, $\overline{\mathbf{E}}$ points along the negative x direction at P (the direction in which a positive point charge would move)

Consider a small element of the rod, dx, at distance x from the origin.



5 for a good diagram (even though most of it is given in the question)

The charge on dx is $dq = \lambda dx$

$\Rightarrow$ Electric field at P due to dx is $$d\overline{\mathbf{E}} = \frac{\lambda dx}{4\pi\varepsilon_0(x+r)^2}\,\hat{\mathbf{i}}$$

The total field is obtained by integrating this over the whole rod: x = 0 to x = -∞.

$$\overline{\mathbf{E}} = \int_0^{-\infty} d\overline{\mathbf{E}} = \left[\frac{\lambda}{4\pi\varepsilon_0} \int_0^{-\infty} \frac{dx}{(x+r)^2} \right] \hat{\mathbf{i}}$$

8 for constructing this argument and deriving the integral

Therefore $$\overline{\mathbf{E}} = \frac{\lambda}{4\pi\varepsilon_0} \left[\frac{-1}{x+r} \right]_0^{-\infty} \hat{\mathbf{i}} \Rightarrow \overline{\mathbf{E}} = \left[\frac{\lambda}{4\pi\varepsilon_0 r} \right] \hat{\mathbf{i}}$$

4 for working out the final answer

Note: One might get a negative answer if one chose the limits of the integral the other way around (from -∞ to 0). In that case we would just take the absolute magnitude - because **WE KNOW FROM THE DIAGRAM** that the electric field points along the +X direction and so is [something positive]$\hat{\mathbf{i}}$.

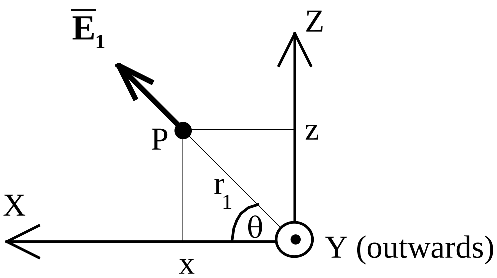
5 **Q5**

Method: Consider each thread separately and resolve the field contributions into their x, y and z components.

~ 4 for method
~ 1 for correct working and answer

use discretion: this is for the better students to show that they have a well-developed understanding

Consider the y-axis thread: Let its field be $\overline{\mathbf{E}}_1$. It has zero y-component.



Its magnitude is $E_1 = \dfrac{\lambda}{2\pi\varepsilon_0 r_1}$ (derived in lectures)

$\lambda = 4$ $\qquad x = 2$ $\qquad z = 2$ $\qquad r_1 = (2^2 + 2^2)^{1/2} = 8^{1/2} = 2\sqrt{2}$.

So $E_1 = \dfrac{1}{\sqrt{2}[\pi\varepsilon_0]}$.

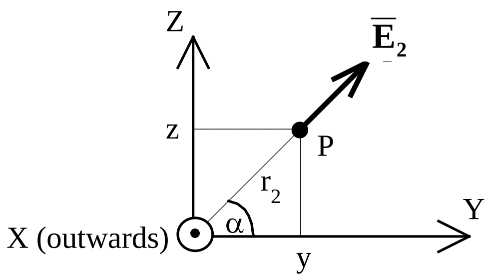
From the diagram, $\overline{\mathbf{E}}_1 = E_1 \cos\theta\, \hat{\mathbf{i}} + 0\, \hat{\mathbf{j}} + E_1 \sin\theta\, \hat{\mathbf{k}}$.

$\cos\theta = \sin\theta = 2/r_1 = 1/\sqrt{2}$

So $$\overline{\mathbf{E}}_1 = \frac{1}{\sqrt{2}[\pi\varepsilon_0]} \left[\frac{1}{\sqrt{2}} \hat{\mathbf{i}} + 0\, \hat{\mathbf{j}} + \frac{1}{\sqrt{2}} \hat{\mathbf{k}} \right] = \frac{1}{2\pi\varepsilon_0} \left[\hat{\mathbf{i}} + 0\, \hat{\mathbf{j}} + \hat{\mathbf{k}} \right]$$

Similarly, consider the x-axis thread:
Let its field be $\overline{\mathbf{E}}_2$.

It has zero x-component. Its magnitude is



$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 r_2}$$

$\lambda = 4$ $y = 2$ $z = 2$ $r_2 = (y^2 + z^2)^{1/2} = 8^{1/2} = 2\sqrt{2}$

So $$E_2 = \frac{1}{\sqrt{2}[\pi\varepsilon_0]}$$

From the diagram, $$\overline{\mathbf{E}}_2 = 0\,\hat{\mathbf{i}} + E_2\cos\alpha\,\hat{\mathbf{j}} + E_2\sin\alpha\,\hat{\mathbf{k}}$$

$\cos\alpha = \sin\alpha = 2/r_2 = 1/\sqrt{2}$

So $$\overline{\mathbf{E}}_2 = \frac{1}{\sqrt{2}[\pi\varepsilon_0]}\left[0\,\hat{\mathbf{i}} + \frac{1}{\sqrt{2}}\hat{\mathbf{j}} + \frac{1}{\sqrt{2}}\hat{\mathbf{k}}\right] = \frac{1}{2\pi\varepsilon_0}\left[0\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right].$$

Adding $\overline{\mathbf{E}}_1$ and $\overline{\mathbf{E}}_2$ we get $$\overline{\mathbf{E}} = \frac{1}{2\pi\varepsilon_0}\left[\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right].$$