ELECTRIC AND MAGNETIC FIELDS
ANSWERS TO WEEK 1 ASSIGNMENT

15

Q1 $\overline{\mathbf{A}} = -10\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\overline{\mathbf{B}} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$$\overline{\mathbf{A}} + \overline{\mathbf{B}} = (-10+4)\hat{\mathbf{i}} + (-5-3)\hat{\mathbf{j}} + (2+2)\hat{\mathbf{k}} = -6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$ 5

$$\overline{\mathbf{A}} + \overline{\mathbf{B}} = (-10-4)\hat{\mathbf{i}} + (-5+3)\hat{\mathbf{j}} + (2-2)\hat{\mathbf{k}} = -14\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$$ 5

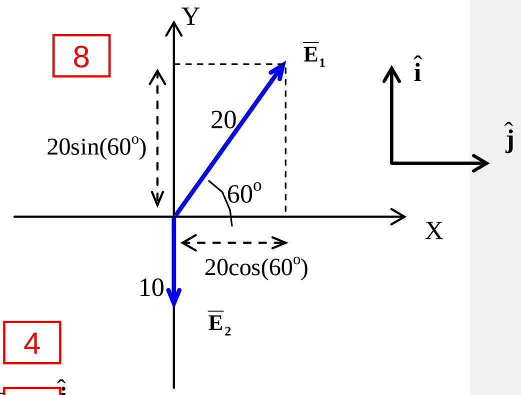
$$4\overline{\mathbf{A}} - 3\overline{\mathbf{B}} = (-40-12)\hat{\mathbf{i}} + (-20+9)\hat{\mathbf{j}} + (8-6)\hat{\mathbf{k}} = -52\hat{\mathbf{i}} - 11\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$ 5

20

Q2

(i) Diagram:

8



(ii) Express vectors in terms of orthogonal unit vectors:

$\overline{\mathbf{E}}_1 = 20\text{Cos}(60^o)\ \hat{\mathbf{i}} + 20\sin(60^o)\ \hat{\mathbf{j}}$

So $\overline{\mathbf{E}}_1 = 10\ \hat{\mathbf{i}} + 17.3\ \hat{\mathbf{j}}$

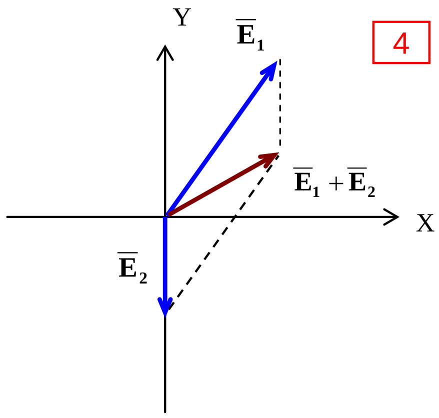
4

$\overline{\mathbf{E}}_2$ only has a Y-component: $\overline{\mathbf{E}}_2 =$ 4 $\hat{\mathbf{j}}$

Adding $\overline{\mathbf{E}}_1$ and $\overline{\mathbf{E}}_2$, we get $\overline{\mathbf{E}}_1 + \overline{\mathbf{E}}_2 = 10\ \hat{\mathbf{i}} + 7.3\ \hat{\mathbf{j}}$

(iii) Illustrate the resultant:

4



15 **Q3** Find a vector of magnitude 14 which is perpendicular to $\overline{\mathbf{A}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$

~ 10 for the method

~ 5 for getting the exact answer

If we divide any vector by its magnitude, we get a UNIT vector with the same direction. Here, we want the magnitude to be 28, so we simply multiply the result by 28. To reverse the direction, we then change the signs of the three components.

Magnitude of $\overline{\mathbf{A}}$ is $A = (3^2 + 2^2 + 6^2)^{1/2} = 7$

$\Rightarrow$ Unit vector in direction of $\overline{\mathbf{A}}$ is $\hat{\mathbf{a}} = \frac{3}{7}\hat{\mathbf{i}} + \frac{2}{7}\hat{\mathbf{j}} - \frac{6}{7}\hat{\mathbf{k}}$

$\Rightarrow$ Vector of magnitude 28, opposite to $\overline{\mathbf{A}}$ is

$-28\hat{\mathbf{a}} = -12\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 24\hat{\mathbf{k}}$

15 **Q4** $\overline{\mathbf{A}} = -3\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$ and $\overline{\mathbf{B}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$.

(i) Dot product: $\overline{\mathbf{A}} \cdot \overline{\mathbf{B}} = (-3)(3) + (-8)(-2) + (7)(5)$ 8 2

(ii) Angle between A and B: By definition, $\overline{\mathbf{A}} \cdot \overline{\mathbf{B}} = AB\cos\theta$

So $(-3^2 + -8^2 + 7^2)^{1/2}(3^2 + -2^2 + 5^2)^{1/2}\text{Cos}\theta = 42$

Therefore $\text{Cos}\theta = 42/68.1 = 0.62 \Rightarrow \theta = 51.9^o$ 7

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15 **Q5**

~ 10 for method

~ 5 for getting the exact answer

Zero marks for using the determinant method

$\overline{\mathbf{P}} \times \overline{\mathbf{E}} = (3\hat{\mathbf{i}} - 5\hat{\mathbf{j}}) \times (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

$= (3\hat{\mathbf{i}} \times 2\hat{\mathbf{i}}) + (3\hat{\mathbf{i}} \times 4\hat{\mathbf{j}}) + (-5\hat{\mathbf{j}} \times 2\hat{\mathbf{i}}) + (-5\hat{\mathbf{j}} \times 4\hat{\mathbf{j}})$

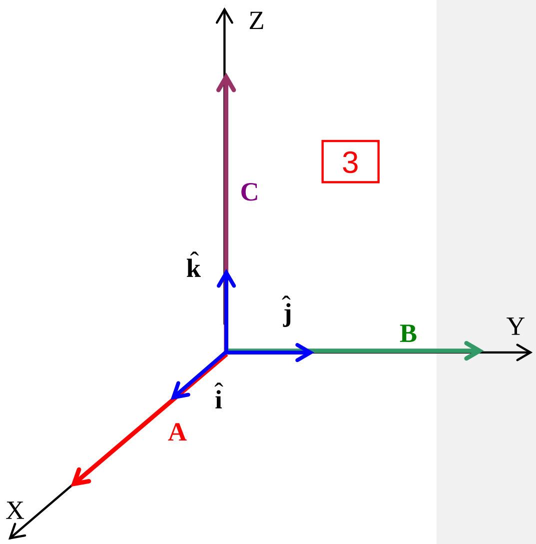
$= 6(\hat{\mathbf{i}} \times \hat{\mathbf{i}}) + 12(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) - 10(\hat{\mathbf{j}} \times \hat{\mathbf{i}}) - 20(\hat{\mathbf{j}} \times \hat{\mathbf{j}})$

Now, $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = 0$ $\quad \hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$ $\quad \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$

Therefore $\overline{\mathbf{P}} \times \overline{\mathbf{E}} = 22\hat{\mathbf{k}}$

$\boxed{15}$ **Q6** $\overline{\mathbf{A}} = 3\hat{\mathbf{i}}$ $\overline{\mathbf{B}} = 4\hat{\mathbf{j}}$ $\overline{\mathbf{C}} = 5\overline{\mathbf{z}}$

(i) Diagram showing the x, y and z axes, the orthogonal unit vectors, and the vectors **A**, **B** and **C**.



(ii)

$\overline{\mathbf{A}} \times \overline{\mathbf{B}} = 12(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) = 12\hat{\mathbf{k}}$ $\boxed{3}$

$\overline{\mathbf{A}} \times \overline{\mathbf{C}} = 15(\hat{\mathbf{i}} \times \hat{\mathbf{k}}) = -15\hat{\mathbf{j}}$ $\boxed{3}$

$\boxed{3}$

$\overline{\mathbf{C}} \times \overline{\mathbf{B}} = 20(\hat{\mathbf{k}} \times \hat{\mathbf{j}}) = -20\hat{\mathbf{i}}$

$\overline{\mathbf{A}} \cdot \overline{\mathbf{B}} = \overline{\mathbf{A}} \cdot \overline{\mathbf{C}} = \overline{\mathbf{C}} \cdot \overline{\mathbf{B}}$ $\boxed{3}$
(they are all perpendicular to each other)

$\boxed{5}$ **Q7** $\overline{\mathbf{A}} = -2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

Vector, $\overline{\mathbf{B}}$, has magnitude $90^{1/2}$, lies in the first quadrant of the x-y plane, and has direction perpendicular to $\overline{\mathbf{A}}$.

Let the required vector be $\overline{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} +$ $\boxed{1}$

$\overline{\mathbf{B}}$ is in the x-y plane $\Rightarrow$ $\boxed{1}$ = 0.

$\overline{\mathbf{A}}$ and $\overline{\mathbf{B}}$ are perpendicular $\Rightarrow$ $\overline{\mathbf{A}} \cdot \overline{\mathbf{B}} = 0$

Mark on method using this as a guide

$\Rightarrow$ $-2B_x + 6B_y = 0$

$\boxed{1}$

$\Rightarrow$ $B_x = 3B_y$

$\overline{\mathbf{B}}$ has magnitude $40^{1/2}$ $\Rightarrow$ $\boxed{1}$

$B_x^2 + B_y^2 = 90$

$\Rightarrow$ $10\,B_y^2 = 90$

$\Rightarrow$ $B_y = \pm 3$ and $B_x = \pm 9$

$\overline{\mathbf{B}}$ is in the first quadrant, so B_x and B_y are positive $\Rightarrow$ $\overline{\mathbf{B}} = 9\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$. $\boxed{1}$