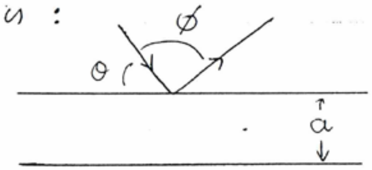
HOMEWORK1 MODEL ANSWERS

(1) The Bragg relation is :

$n\lambda = 2a \sin\theta$



$\phi = 50°$

$2\theta + \phi = 180°$
$\Rightarrow \theta = 65°$

54 eV corresponds to 8.64×10^{-18} J, $m_e = 9.11 \times 10^{-31}$
using $KE = p^2/2m$, $p = \sqrt{2m \times (KE)} = 3.97 \times 10^{-24}$ kg m/s

now $\frac{h}{p} = \lambda = 1.67 \times 10^{-10}$ m

From Bragg relation, $a = \frac{n\lambda}{2\sin\theta} = n \times 9.21 \times 10^{-11}$ m

Taking smallest separation, $n = 1 \Rightarrow a = .092$ nm.

b) for momenta p_1, p_2 with corresponding wavelengths λ_1, λ_2, then

$$\lambda_1/\lambda_2 = \frac{p_2}{p_1} = \sqrt{\frac{E_2}{E_1}}$$

$\therefore$ at 70 eV , $\lambda = 1.67 \times 10^{-10}\sqrt{\frac{54}{70}} = 1.47 \times 10^{-10}$ m

Then, from the Bragg relation,

$$\sin\theta = \frac{1.47 \times 10^{-10}}{2 \times 9.21 \times 10^{-11}} = .798$$

$\therefore \theta = 52.9°$ and $\phi = 74.1°$

(1) CONTINUED

At 35 eV , $\lambda = 1.67 \times 10^{-10}\sqrt{\frac{54}{35}} = 2.07 \times 10^{-10}$

and $\sin\theta = 1.126 > 1$!

Not acceptable so no solution at this energy and no maximum.

c) For neutrons to produce a peak at the same angle, they must have the same wavelength as 54 eV electrons and hence the same momentum

$$\frac{E(\text{neutron})}{E(\text{electron})} = \frac{p^2(\text{neutron})}{p^2(\text{electron})} \times \frac{m_e}{m_n} = \frac{m_e}{m_n}$$

so neutron energy = $54\text{ eV} \times 5.46 \times 10^{-4}$
$= .029$ eV

Q.2
If the wave function is multiplied by a constant, N, we require, for normalization, that

$$|N|^2 \int_{-1}^{1} U^2(x)\,dx = 1$$

(a) $N^2 \int_{-1}^{1} dx = N^2 [x]_{-1}^{+1} = 1 \quad \therefore N^2 = 1/2$

$U(x) = 1/\sqrt{2}$

(b) $N^2 \int_{-1}^{1} 4x^2\,dx = 1 \quad \therefore 4N^2[x^3/3]_{-1}^{1} = 1 \Rightarrow N^2 = 3/8$

$U(x) = \sqrt{3/2}\, x$

(c) $N^2 \int_{-1}^{1} x^4\,dx = 1 \quad \therefore N^2 = 5/2$

and $U(x) = \sqrt{5/2}\, x^2$

3) TISE $\quad H(x)U(x) = E\,U(x)$

$$= -\frac{\hbar^2}{2m}\frac{d^2U}{dx^2} + V(x)U(x) = E\,U(x)$$

$$U(x) = e^{-\alpha x^2/2}$$

$$\frac{d^2U}{dx^2} = -\alpha e^{-\alpha x^2/2} + \alpha^2 x^2 e^{-\alpha x^2/2}$$

in TISE, $\quad \frac{\hbar^2\alpha}{2m} e^{-\alpha x^2/2} - \frac{\hbar^2\alpha^2}{2m} x^2 e^{-\alpha x^2/2} + \frac{m\omega^2}{2} x^2 e^{-\alpha x^2/2} = E\,U(x)$

if $\alpha = \frac{m\omega}{\hbar}$,

we have $\quad \frac{\hbar\omega}{2} e^{-\alpha x^2/2} - \hbar\frac{m\omega^2}{2}x^2 e^{-\alpha x^2/2} + \frac{m\omega^2}{2}x^2 e^{-\alpha x^2/2} = E\,U(x)$

(1) ie $\quad \frac{\hbar\omega}{2} e^{-\alpha x^2/2} = \frac{\hbar\omega}{2} U(x) = E\,U(x)$

(4) so, by comparison, $E = \frac{1}{2}\hbar\omega$.

12 MARKS

Q 4. Time to fall a distance H from rest is $\sqrt{\frac{2H}{g}}$.

Neglect uncertainty in the vertical initial conditions. Horizontally, there is an uncertainty in initial position Δx_0 and initial momentum Δp_0 related by $\Delta x_0 \Delta p_0 \geq \hbar/2$. Take the equality to minimise this uncertainty.

The momentum Δp_x results in a displacement $\frac{\Delta p_x}{m}\sqrt{\frac{2H}{g}}$ on the ground.

$$\therefore \quad X \simeq \Delta x_0 + \frac{\Delta p_x}{m}\sqrt{\frac{2H}{g}}$$

(6)
$$X \simeq \Delta x_0 + \frac{\hbar}{2m\Delta x_0}\sqrt{\frac{2H}{g}}$$

for minimum X

$$\frac{dX}{d(\Delta x_0)} = \frac{d}{d(\Delta x_0)}\left(\Delta x_0 + \frac{\hbar}{2m\Delta x_0}\sqrt{\frac{2H}{g}}\right) = 0$$

when $\quad \Delta x_0 = \left(\frac{\hbar}{2m}\sqrt{\frac{2H}{g}}\right)^{1/2}$

(6) and $\quad X = \sqrt{\frac{2\hbar}{m}\sqrt{\frac{2H}{g}}}$

for $m = 10^{-6}\,g$, $H = 2\,m$ the spread X

(2)
$$X = \sqrt{\frac{2\hbar}{m}\sqrt{\frac{2H}{g}}} = 3.67 \times 10^{-13}\ M$$