EMF
REVISION NOTES

These notes summarise the principal topics and the most important equations met during the course. They should be regarded only as an aid to revision and NOT as a complete condensation of the course material.

The EMF paper will consist of two section, Section A and Section B.

Section A

This section is *compulsory* and is worth 40% of the total marks in the exam. It will consist of short questions (about 6) which will cover the main aspects of the course syllabus but not in great depth. Typically in this section you will be asked to state e.g Gauss's law for the electric field and explain the symbols involved and how one might use the law in practice. There may also be short calculations in the questions of section A.

Section B

In this section students will have a choice of questions..typically 2 out of 4 questions to answer. Each question is worth 30% of the total exam marks. The nature of these questions is very similar to those in previous years exams in EMF. Thus it is good practice to look at these past papers for guidance. On the EMF homepage, there are past papers and model answers going back several years.

GENERAL ADVICE

1. **REMEMBER and UNDERSTAND the PHYSICAL MEANINGS of the essential principles. For all of the fundamental relationships you should be able to STATE the law by writing**

the EQUATION, to define all the symbols and to EXPLAIN what the equation means using WORDS and DIAGRAM(S).

2. **Learn the METHODS by which the fundamental principles are applied to solve problems:**

   → **Regard the problems done in the lectures, exercise classes and assignments as EXAMPLES of how to apply the basic laws. Try to see how the METHOD is always the same even for problems which look different in their details.**

   e.g., All the examples we did using Gauss's Law are really the same.

   All the examples we did using Ampere's Law are really the same.

   etc.

3. **The examination will test your knowledge and understanding of the basic ideas and also your ability to APPLY them to problems similar to those met during the course.**

4. **Manage your time well in the exam. Don't spend too long on the last couple of marks if easy marks are to had elsewhere.**

5. **When answering exam questions, try to be clear about what you are doing. The ideal answer is a mixture of EQUATIONS, WORDS and DIAGRAM(S).**

6. **Draw and clearly label DIAGRAM(S) when doing questions:**

   (i) it helps you to visualise the problem and keeps you on the right track in finding the solution;

   (ii) it proves to the marker that you know what you are doing.

7. **When necessary, think in THREE DIMENSIONS, and be prepared to shift your spatial point of view if needed.**

8. Remember the laws of VECTOR ALGEBRA, especially the DOT and CROSS products.

9. Distinguish between vector and scalar quantities (standard method = put a bar over a vector).

10. Don't rely on these notes or any photocopied handouts for final revision - if it's not in your own handwriting you probably won't be able to remember it.

VECTORS AND SCALARS

Scalar: Magnitude only
Vector: Magnitude and direction

Examples of scalars		**Examples of vectors**	
Electric flux	ϕ	Force	$\overline{F}$
Magnetic flux	Ψ	Velocity	$\overline{v}$
Electric potential	V	Electric field	$\overline{E}$
Capacitance	C	Magnetic field	$\overline{B}$
Inductance	L	Dipole moment	$\overline{P}$ or $\overline{\mu}$
Dot product	$\overline{A}.\overline{B}$	Cross product	$\overline{A} \times \overline{B}$

VECTOR NOTATION

Written notes : If it is a vector, put a bar over it
Printed material : Boldface letters are usually used for vectors

Note: In this handout, bars will be used to denote vectors as a reminder that this is what you must do in written work (e.g., the exam.)

Unit vector: Magnitude = 1 Symbolised by ^

Orthogonal unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ point along the three axes.

VECTOR ADDITION

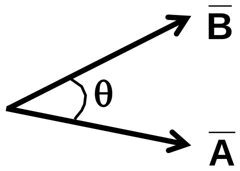
(i) Parallelogram law
(ii) Decompose vectors into their x, y, z components
(iii) Do NOT add vectors as scalars

VECTOR MULTIPLICATION

(i) By a scalar: $n\overline{A}$ has magnitude $= nA$
direction = same as that of $\overline{A}$

(ii) **DOT Product:** $\overline{A} \cdot \overline{B}$ is a SCALAR

$$\overline{A} \cdot \overline{B} = AB\cos\theta$$

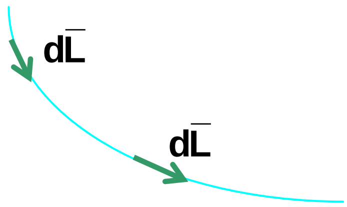


(iii) CROSS Product: $\overline{A} \times \overline{B}$ **is a VECTOR**

Magnitude: $|\overline{A} \times \overline{B}| = AB\sin\theta$

Direction: **Perpendicular to** $\overline{A}$ **and** $\overline{B}$ **given by the Right Hand Rule**

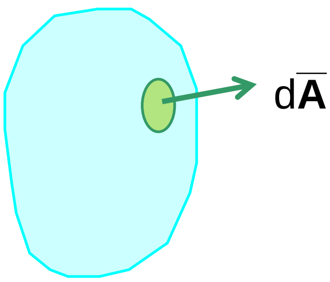
LINE AND SURFACE ELEMENT VECTORS



(I) Any path can be divided into many small LINE ELEMENTS

At any point, $d\overline{L}$ **has** **Magnitude = length dL**
Direction tangential to the path

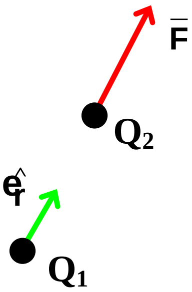
(ii) Any surface can be divided into may small SURFACE ELEMENTS



For any small patch of area ds, the NORMAL VECTOR is $d\overline{A}$

$d\overline{A}$ **has magnitude = area dA**
direction perpendicular to ds pointing outwards

THE ELECTRIC FORCE $\overline{F} = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}\hat{r}$ **Coulomb's Law**



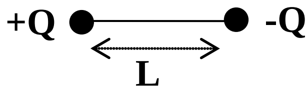
THE ELECTRIC FIELD $\overline{E} = \dfrac{\overline{F}}{Q}$ **Force per unit charge**

$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$ **at distance r from a point charge Q**

PRINCIPLE OF SUPERPOSITION **Electric fields and forces add as vectors**

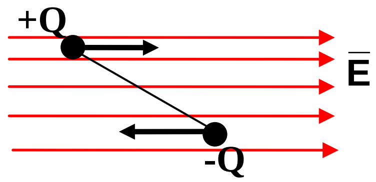
Examples:
- **Two-dimensional examples involving point charges in the x-y plane**
- **Field on the axis of a line of charge**

ELECTRIC DIPOLE



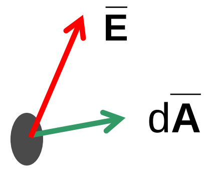
Dipole moment vector: $\bar{P}$ has
Magnitude = QL
Direction from -Q to +Q

Torque on a dipole due to $\bar{E}$ is $\bar{P} \times \bar{E}$



ELECTRIC FLUX

Flux through small flat area ds is $d\phi = \bar{E} \cdot d\bar{A}$



i.e., Flux $\equiv$ (Field)(Area)

GAUSS'S LAW

Using Coulomb's Law and the concept of electric flux, we derived Gauss's Law

$$\Phi = \oint \bar{E} \cdot d\bar{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Know also how to
- express it in words
- explain its physical meaning
- apply it to solve problems

Using Gauss's Law:

When? When you are given some distribution of charge and you need to find the electric field.

How?
1. Draw a diagram showing the electric field pattern
2. Choose the best Gaussian surface to make the integral easy:

   i.e., make $\bar{E}$ and $d\bar{s}$ either parallel or perpendicular

3. Work out $\Phi = \oint \bar{E} \cdot d\bar{A}$

4. Decide how much charge, Q_{enc}, is *inside* the Gaussian surface.

5. Set $\Phi = Q_{enc}/\varepsilon_0$ and rearrange to find E.

Examples: **Point charge, line of charge, plane of charge, sphere of charge, etc.**

Gaussian surface is usually either a cylinder or a sphere. Questions often have a number of parts (e.g., find E at different radii): in these cases, the surface integral is usually of the same form but the enclosed charge may be different for the different regions.

Spherical symmetry $\Rightarrow$ $\bar{E}$ at any point is due only to the charge inside its radius, and is the same as if all that charge were concentrated at the centre [easily proved using Gauss's Law].

CONDUCTORS IN ELECTRIC FIELDS

Electrostatic equilibrium $\Rightarrow$ $\bar{E} = 0$ inside a perfect conductor

$\bar{E}$ is perpendicular to the surface of a perfect conductor

Gauss's Law $\Rightarrow$ All excess charge lies at the surface of a perfect conductor

ELECTRIC POTENTIAL, V

V at a point = PE which a charge Q *would* have at that point divided by Q.

i.e., V = U/Q $\equiv$ PE per unit charge **SI units: Volts**

Relationship between $\bar{E}$ and V:

Potential difference is the line integral of the electric field

$$V_a - V_b = -\int_a^b \bar{E} \cdot d\bar{L}$$

Electric field $\equiv$ Potential gradient

$$\bar{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right] = -\nabla V$$

Zero of potential: **Defined arbitrarily - often at infinity or at the surface of a conductor.**

THE ELECTRIC FIELD IS CONSERVATIVE

The work done in moving a charge is independent of the path taken: $\oint \overline{E} \cdot d\overline{L} = 0$

$\Rightarrow$ $\overline{E}$ is zero inside a closed empty cavity in a perfect conductor

ELECTRIC POTENTIAL ENERGY CALCULATIONS

Method 1: Integrate the electric field

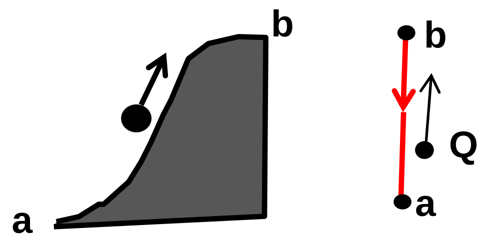
1. Find $\overline{E}$ if it is not given (e.g., use Gauss's Law)

2. Choose the position of zero V (if it is not given)

3. Put $|\Delta V| = \left| \int_a^b \overline{E} \cdot d\overline{L} \right|$ Forget about the sign: just find the magnitude of ΔV

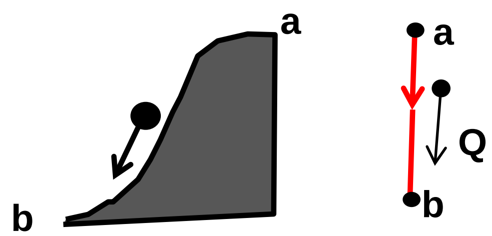
4. Use common sense and the definition of V to determine the sign of ΔV :

If you would need to PUSH a positive charge against $\overline{E}$ to go from a to b then



$V_b > V_a$ Analogy: pushing a ball uphill

If a positive charge would be pulled by $\overline{E}$ from P_1 to P_2 then



$V_a > V_b$ Analogy: A ball rolling downhill

Examples: V at distance r from a point charge $V = \dfrac{Q}{4\pi\varepsilon_0 r}$

ΔV due to a plane of charge (close analogy with a uniform gravitational field)

ΔV due to a long cylinder of charge

Method 2: Use the principle of superposition

1. Divide the charge distribution into many small elements.

2. Regard each element as a point charge and find its contribution to the potential using $V = \dfrac{Q}{4\pi\varepsilon_0 r}$.

3. Integrate over the whole charge distribution to find the total potential.

EQUIPOTENTIAL SURFACE V is the same everywhere

$\Rightarrow$ $\bar{E}$ points perpendicular to an equipotential surface (e.g., the surface of a conductor).

ELECTRIC ENERGY

For a system of n point charges $U_{tot} = \dfrac{1}{2}\sum_{i=1}^{n} Q_i V_i$

where V_i = potential at the position of charge Q_i due to the combined effects of all the other charges.

Energy of a charged conductor: $U = \dfrac{1}{2}QV$

Examples:
- Conducting sphere
- Parallel plate capacitor
- Etc.

ENERGY DENSITY OF THE ELECTRIC FIELD $u = \frac{1}{2}\varepsilon_0 E^2$

CAPACITANCE, C

In general $V \propto Q$ $C = Q/V$

$\Rightarrow$ Energy of a charged conductor is $U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

Finding C:

1. Imagine $\pm Q$ placed on conductors.
2. Find $\bar{E}$ (e.g., use Gauss's Law).
3. Find $|\Delta V|$ (never mind the sign).
4. Put $C = Q/V$
   - Q cancels out
   - C depends only on the size, shape, separation of the conductors

Examples:
- Conducting sphere
- Parallel plate capacitor
- Spherical capacitor
- Cylindrical capacitor

Capacitors in parallel: $C_{tot} = C_1 + C_2$ (V is same for both)

Capacitors in series: $\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2}$ (Q is same for both)

DIELECTRICS AND POLARISATION

$$\bar{E}_{tot} = \bar{E}_o - \bar{E}_p = \frac{\bar{E}_o}{K}$$

This equation defines the **DIELECTRIC CONSTANT**

$K \;(\geq 1)$

When the medium is not a vacuum, simply replace ε_0 with $\kappa\varepsilon_0$.

ELECTRIC CURRENT

$I = dQ/dt$ $I \propto E \Rightarrow I \propto \Delta V$

Resistivity and resistance: $R = \rho\frac{L}{A}$ L = length; A = Area

Ohm's Law: $I = \frac{\Delta V}{R}$

ELECTROMOTIVE FORCE, $\mathcal{E}$

$\mathcal{E}$ = PE gained by one Coulomb of charge in passing through source of emf (analogy of water pump working in the Earth's gravitational field)

$\mathcal{E} = U/Q$ Energy per unit charge

$\Rightarrow$ Units are same as Potential, Volts

Note: emf is NOT a force

ELECTRIC POWER

$$P = \frac{dU}{dt} = \mathcal{E} = \mathcal{E}I$$

Power dissipated (as heat) in a resistor $P = I^2R = V^2/R$

KIRCHHOFF'S LAWS

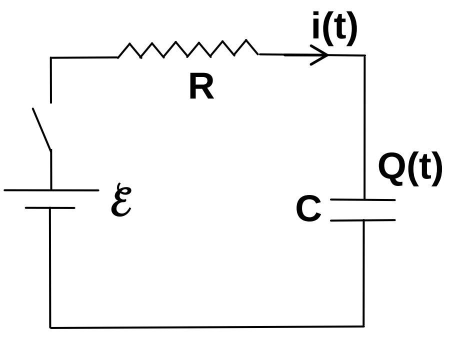
Voltage Law: For any closed loop in a circuit, the sum of all emfs and potential drops is zero.

Current Law: The sum of all currents flowing into a node is zero.

STEP RESPONSE OF THE RC CIRCUIT

(Example of a first-order linear system)

Switch closed at t = 0
Capacitor charges up

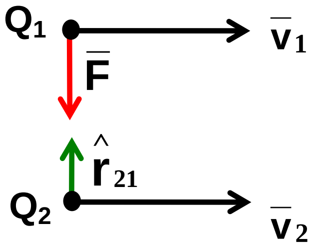


1. Use KVL to derive differential equation for Q.
2. Separate variables, Q on left, t on right
3. Integrate and use initial conditions to find constant of integration $\rightarrow$ find Q(t).

THE MAGNETIC FORCE

Caused by charges in MOTION (i.e., currents)

$$\bar{F} = \frac{\mu_0 Q_1 Q_2}{4\pi r^2}(v_1 v_2)\hat{r}_{21}$$



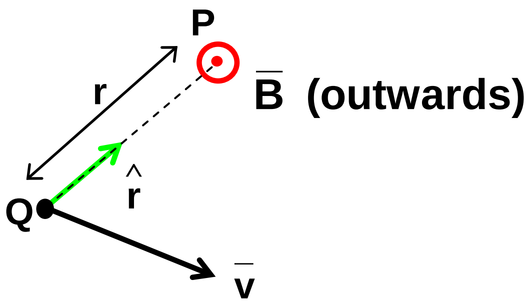
Note: **Magnetic force is perpendicular to v $\Rightarrow$ it doesn't change the speed of the particle, only its DIRECTION**

THE MAGNETIC FIELD

Defined by $\bar{F} = Q(\bar{v} \times \bar{B})$

Magnetic field at P due to moving Q is

$$\bar{B} = \frac{\mu_0 Q}{4\pi r^2}(\bar{v} \times \hat{r})$$



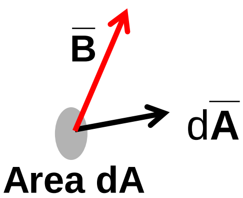
MAGNETIC FIELD LINES

Example of charge moving OUT OF PAPER $\rightarrow$ magnetic field lines form CLOSED LOOPS

MAGNETIC FLUX

Flux through small flat area ds is $d\Psi = \bar{B}.\bar{ds}$

i.e., Flux $\equiv$ (Field)(Area)



GAUSS'S LAW FOR THE MAGNETIC FIELD $\Psi = \oint \bar{B} \cdot d\bar{A} = 0$

(Magnetic field lines form closed loops; there are no magnetic monopoles)

THE LORENTZ FORCE

If Q moves with velocity $\overline{v}$ in an electric field $\overline{E}$ and a magnetic field $\overline{B}$ then force on it is

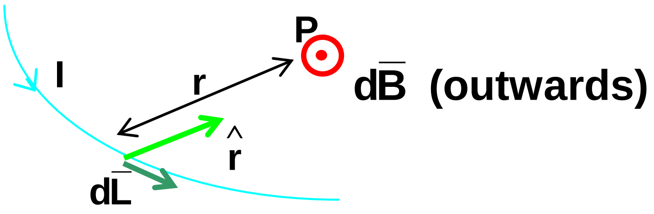
$$\overline{F} = Q\left[\overline{E} + (\overline{v} \times \overline{B})\right]$$

Examples: **Velocity selector**
Magnetic field only → circular or spiral motion
Hall effect: Conductor in magnetic field
- **→ Magnetic force on charges**
- **→ Separation of charges**
- **→ Transverse electric field (direction gives sign of carriers)**
- **→ ΔV across sides ∝ no. density of the carriers**

BIOT-SAVART LAW

Gives $\overline{B}$ due to a CURRENT- CARRYING ELEMENT

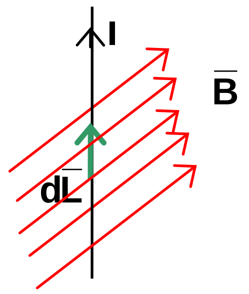
$$d\overline{B} = \frac{\mu_0 I}{4\pi r^2}(d\overline{L} \times \hat{r})$$



To find the total field at P, integrate over the whole length of the wire.

FORCE ON A CURRENT- CARRYING WIRE IN A MAGNETIC FIELD

$$d\overline{F} = dQ(\overline{v} \times \overline{B}) \Rightarrow d\overline{F} = I(d\overline{L} \times \overline{B})$$



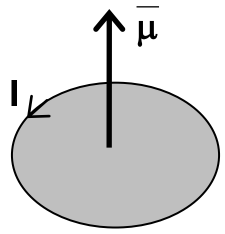
If $\overline{B} \perp$ wire, then dF = BIdL

⇒ Force between two parallel wires is $F = \frac{\mu_0 I_1 I_2}{2\pi r}$

(basis of the definition of the Ampere).

MAGNETIC DIPOLE (≡ CURRENT- CARRYING LOOP)

DIPOLE MOMENT VECTOR $\bar{\mu}$



Magnitude = nIA = (No. of turns)(Current)(Area)
Direction = ⊥ to the plane of the loop, given by the right hand rule

TORQUE on a magnetic dipole due to external $\bar{B}$: $\bar{\tau} = \bar{\mu} \times \bar{B}$

RELATIVE PERMEABILITY

If the medium is not a vacuum, then replace μ_0 with $\mu_0\mu_r$

$\mu_r \approx 1$ for most materials

AMPERE'S LAW

Relates the magnetic field to the current distribution that produces it

$$\oint \bar{B} \cdot d\bar{L} = \mu_o I_{enc}$$

Using Ampere's Law:

When? When you are given some distribution of charge and you want to find the magnetic field

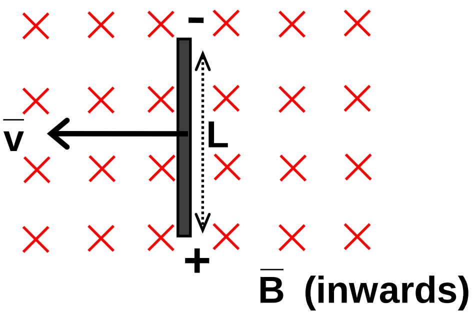
How?
1. Draw a diagram showing the magnetic field pattern
2. Choose an imaginary closed path to make the line integral easy

   i.e., make $\bar{B}$ and $d\bar{L}$ either parallel or perpendicular

   View along the axis (current flowing out of the page, so that you can draw the path in the plane of the page.
3. Work out $\oint \bar{B} \cdot d\bar{L}$
4. Decide how much current, I_{enc}, is flowing through the loop.
5. Equate the results of 3 and 4 and rearrange to find B.

Examples:
- Long thin wire: $B = \mu_o I/(2\pi r)$
- Long solid cylinder (similar to wire)
- Long solenoid: $B = \mu_o nI$

ELECTROMAGNETIC INDUCTION



Changing $\bar{B}$ → Induced $\bar{E}$

Introduced through MOTIONAL emf :

$$\mathcal{E} = vBL$$

→ $\mathcal{E}$ = **Rate of sweeping out of magnetic flux:**

$$\mathcal{E} = -\frac{d\Psi}{dt} \quad \text{or} \quad \oint \bar{E} \cdot d\bar{L} = -\frac{d\Psi}{dt}$$ **FARADAY'S LAW**

The emf induced around a closed loop = - Rate of change of magnetic flux through the loop

Negative sign ≡ LENZ'S LAW: The induced emf OPPOSES the CHANGE in $\bar{B}$ that produces it (i.e., it tries to keep $\bar{B}$ constant).

INDUCTANCE

Changing current in one circuit
→ **changing $\bar{B}$**
→ **changing Ψ**
→ **induced $\bar{E}$ in another circuit (Mutual Inductance)**
in the same circuit (Self Inductance)

Inductance = Flux/Current

MUTUAL INDUCTANCE

$$M = \frac{\Psi_{21}}{I_1}$$

Ψ_{21} = Flux through circuit 2 due to current I_1 in circuit 1

$$\mathcal{E}_2 = -M\frac{dI_1}{dt}$$

SELF INDUCTANCE

$$L = \frac{\Psi}{I}$$

Ψ = Flux through circuit due to its own current

$$\mathcal{E} = -L\frac{dI}{dt}$$

Finding M or L:

1. Assume current I flows in the circuit (L) or in one of the circuits (M)
2. Find $\overline{B}$ (e.g., using Ampere's Law)
3. Find Ψ, the flux through the (other) circuit
4. Put L or M = Ψ/I. I will cancel out.
   Inductance depends only on the size, shape, no. of turns, etc.

ENERGY STORAGE IN INDUCTORS

Energy stored = amount of work which must be done in order to increase the current from 0 to I against the opposing (back) emf induced by the changing current.

$$U = \frac{1}{2}LI^2$$

ENERGY DENSITY OF THE MAGNETIC FIELD

$$u = \frac{1}{2}\frac{B^2}{\mu_0}$$

[SI units: J m^{-3}]

(Derived using example of solenoid)

FINDING MAGNETIC ENERGY

1. Find B as a function of position
2. Hence find u
3. Define a suitable VOLUME ELEMENT and integrate u d(Volume) to find U_{tot}.

SUMMARY OF MAXWELL'S EQUATIONS (IN INTEGRAL FORM)

① $$\oint \overline{E} \cdot d\overline{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$ **Gauss's Law for the Electric Field**

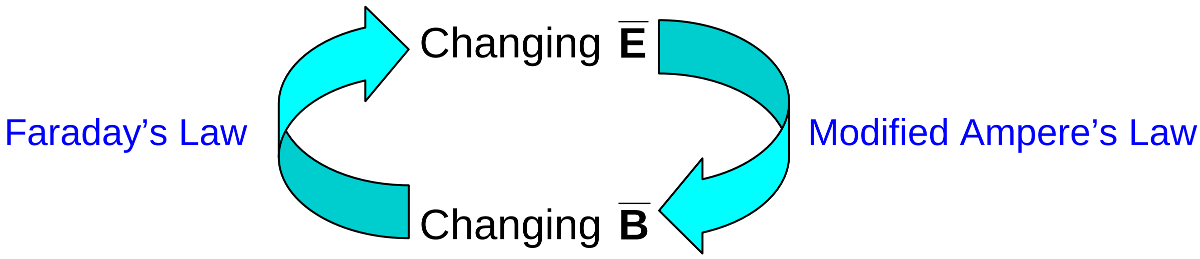
② $$\oint \overline{B} \cdot d\overline{A} = 0$$ **Gauss's Law for the Magnetic Field**

③ $$\oint \overline{E} \cdot d\overline{L} = -\frac{d\Psi}{dt}$$ **Faraday's Law of Induction**

④ $$\oint \overline{B} \cdot d\overline{L} = \mu_o I + \mu_o \varepsilon_o \frac{d\Phi}{dt}$$ **Maxwell's modification of Ampere's Law**

NB: ③ $\Rightarrow$ Changing $\overline{\mathbf{B}}$ generates $\overline{\mathbf{E}}$

④ $\Rightarrow$ Changing $\overline{\mathbf{E}}$ generates $\overline{\mathbf{B}}$



$\Rightarrow$ **OSCILLATION OF ENERGY BETWEEN THE ELECTRIC AND MAGNETIC FIELDS**

$\Rightarrow$ **ELECTROMAGNETIC WAVES EXIST**