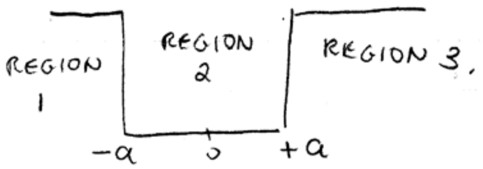
[HANDOUT] SUMMARY OF

FINITE QUANTUM WELL



REGION ① $U_1(x) = C e^{px}$
$U_1'(x) = pC e^{px}$

REGION ③ $U_3(x) = D e^{-px}$
$U_3'(x) = -pD e^{-px}$

after we drop part which 'blows up'

REGION ② $U_2(x) = A \cos kx + B \sin kx$
$U_2'(x) = -Ak \sin kx + Bk \cos kx$.

GET 4 SIMULTANEOUS EQUATIONS BY matching at $x=-a, x=+$

(i) $D e^{-pa} = A \cos ka + B \sin ka$ $\quad U_3(a) = U_2(a)$
(ii) $-pD e^{-pa} = -Ak \sin ka + Bk \cos ka$ $\quad U_3'(a) = U_2'(a)$

(iii) $C e^{-pa} = A \cos ka - B \sin ka$ $\quad U_1(-a) = U_2(-a)$
(iv) $pC e^{-pa} = +Ak \sin ka + Bk \cos ka$ $\quad U_1'(-a) = U_2'(-a)$
ie we used $\sin(-ka) = -\sin(ka)$

Now,

$$\frac{(ii)}{(i)} = \frac{p}{k} = \frac{A \sin ka - B \cos ka}{A \cos ka + B \sin ka}$$

$$= \frac{(iv)}{(iii)} = \frac{p}{k} = \frac{A \sin ka + B \cos ka}{A \cos ka - B \sin ka}.$$

SO
$$\left(\frac{A \sin ka - B \cos ka}{A \cos ka + B \sin ka}\right) = \left(\frac{A \sin ka + B \cos ka}{A \cos ka - B \sin ka}\right) \Rightarrow AB = 0$$

Can see by inspection! Or, try $\frac{\alpha - \beta}{\alpha' + \beta'} = \frac{\alpha + \beta}{\alpha' - \beta'}$

easy to show it implies $\alpha\beta' + \alpha'\beta = 0 = AB(\sin^2 ka + \cos^2 ka$
$\Longrightarrow AB = 0$ .. Hence, either

$p/k = \tan ka$ $(B=0)$ or $p/k = -\cot ka$ $(A=0)$