RELATIVITY MTH6132

PROBLEM SET 6

Please write your name and student number at the top of your assignment and staple all the pages together.

1*) Let $S_b{}^c$ denote a $(1,1)$ tensor.

(i) Give the formula for the covariant derivative $\nabla_a S_b{}^c$ in terms of the connection coefficients.

(ii) Show that

$$\nabla_a \delta_b{}^c = 0.$$

(iii) Show that if the dimension of the manifold is 4, then $\delta_a{}^a = 4$.

2) If $R^a{}_{bcd}$ is a tensor of type $(1,3)$, show that its contraction given by $R_{bd} = R^a{}_{bad}$ is a tensor of type $(0,2)$.

3*) Starting with the covariant derivative of a contravariant vector and assuming that the covariant derivatives obey the ususal Leibnitz rule for differentiation of products and that the covariant derivative of a scalar is the partial derivative, prove that the covariant derivative of a covariant vector V_i is given by:

$$\nabla_j V_i = \partial_j V_i - \Gamma^k{}_{ij} V_k$$

[*Hint: Let the scalar* $\Phi = V_a W^a$ *and recall that* $\nabla_b \Phi = \partial_b \Phi$ *for scalars. Expand and substitute for* $\partial_b W^a$ *from the definition of* $\nabla_b W^a$.]

4*) Consider the two-dimensional space given by

$$ds^2 = e^y dx^2 + e^x dy^2.$$

(i) Calculate the covariant and contravariant components of the metric tensor for this spacetime.

(ii) Employ the formula for the Christoffel symbols (connection) given in the notes to calculate the components $\Gamma^1{}_{11}$, $\Gamma^1{}_{12}$ and $\Gamma^2{}_{11}$ of the connection.

[Note the identification $(x, y) \to (x^1, x^2)$ is used here].

To be placed in the BLUE BOX on 2nd floor of the Maths building by Wednesday 23th November.

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