RELATIVITY MTH6132

PROBLEM SET 3

HAND IN ONLY THE STARRED QUESTIONS.

Write your name and student number at the top of your assignment and staple all the pages together.

1*) In an inertial frame two events occur simultaneously at a distance of $3m$ apart. In a frame moving with resect to the laboratory frame, one event occurs later than the other by $10^{-8}s$. By what spatial distance are the two events separated in the moving frame?

2) Consider the 4-vectors $\bar{A} = (A^0, 0, 2, 0)$ and $\bar{B} = (3, 0, B^2, 0)$, where the components A^0 and B^2 are real constants. Assuming that $\bar{A}$ is a unit spacelike vector, find A^0. Hence find B^2 if $\bar{A}$ and $\bar{B}$ are orthogonal.

3*) Consider two timelike 4-vectors, $\bar{A} = (A^0, A^1, 0, 0)$ and $\bar{B} = (B^0, B^1, 0, 0)$, where the components A^0, A^1, B^0, B^1 are all positive quantities. Show that the sum of the 4-vectors $\bar{A}$ and $\bar{B}$ can never be null.

4) Consider two inertial frames, F and F', in a standard configuration. Consider a 4-vector whose components in frame F are given by $\bar{A} = (A^0, A^1, A^2, A^3)$. Write down the norm of this vector and show that it remains invariant as we go to frame F'.

5*) The 4–velocity $\overline{U} = (U^0, U^1, U^2, U^3)$ corresponds to the 3–velocity, $\underline{v}$, in the sense that $\overline{U} = \gamma(v)(1, \underline{v})$. Express
(1) U^0 in terms of $|\underline{v}|$
(2) U^α in terms of $\underline{v}$, where α represents the spatial components and takes values ($\alpha = 1, 2, 3$)
(3) U^0 in terms of U^α
(4) $\frac{d}{d\tau}$ in terms of $\frac{d}{dt}$ and $\underline{v}$, where τ is proper time
(5) v^α in terms of U^α
(6) $|\underline{v}|$ in terms of U^0

To be handed in on Wednesday 26th October by 6pm in the blue box in the second floor of the School of Mathematical Sciences.

Dr. Juan A. Valiente Kroon (G56)