

3C43 LASERS AND MODERN OPTICS

Problem sheet 4 – Non-linear and Electro optics

(Answers to be handed in on Tuesday, 12th March 2002)

Question 1.

Show that a plane polarised beam of light may be considered to be composed of two oppositely-oriented, circularly polarised components. [4]

Hence show that the rotation of the plane of polarisation in an optically-active medium of thickness d is $(n_r - n_l)\pi d / \lambda_0$, where n_r and n_l are the refractive indices for right and left circularly polarised light respectively. [4]

The specific rotation of quartz is 29.7 deg/mm at $\lambda_0 = 508.6\text{nm}$; calculate the $|n_r - n_l|$. [2]

Question 2.

The principal refractive indices for quartz are $n_e = 1.55336$ and $n_o = 1.54425$.

- Calculate the thickness of a quarter-wave plate for sodium light at $\lambda = 589\text{nm}$. [2]
- Describe how such a plate may be made more robust. [2]
- Calculate the thickness of a calcite half-wave plate, for which $n_e = 1.486$ and $n_o = 1.658$ at the same wavelength. [2]

Show that if linearly polarised light is incident on a half-wave plate such that the plane of polarisation of the light makes an angle θ with the fast-axis of the wave-plate, the plane of polarisation of the light emerging from the wave-plate is rotated through an angle 2θ . [4]

A half-wave plate is placed between a polariser and analyser, which have parallel transmission axes, such that the angle between the transmission axes and the fast axis of the half-wave plate is θ .

If the light incident on the polariser from the left is circularly polarised and has intensity I_0 , show that the intensity of the light emerging to the right of the analyser is

$$I = \frac{I_0}{4}(1 + \cos 4\theta) \quad [5]$$

Question 3.

Write down the equation defining the *linear electro-optic coefficient*, r and hence show that the half-wave voltage of a longitudinal electro-optic modulator is

$$V_{\pi} = \frac{\lambda_0}{2r n_0^3}$$

where n_0 is the refractive index at wavelength λ_0 . [3]

Describe, with reference to a suitable diagram, how such a modulator can be used to control the intensity of a beam of light. [4]

Show that the transmitted intensity is $I = I_0 \sin^2 \frac{\pi}{2} \frac{V}{V_{\pi}}$, where I_0 is the incident intensity. [4]

It is desired to use a longitudinal electro-optic modulator to impose a time-dependence

$$I(t) = \frac{I_0}{2} (1 + \cos 2\omega t)$$

on a laser beam, . Show that the required time-dependent voltage across the modulator is

$$V(t) = V_{\pi} \left(1 + \frac{2\omega t}{\pi} \right) \quad [4]$$

(you may assume the identity: $\cos 2\theta = 2\cos^2 \theta - 1$)