3C43 LASERS AND MODERN OPTICS

Problem sheet 3 – Gaussian Beams

(Answers to be handed in on Tuesday, 26th February 2002)

Question 1.

Write down an expression for the spatial variation of the electric field in a spherical-wave solution of the wave-equation governing the propagation of light. [2]

Show, detailing the approximations used, how the spherical-wave solution can be modified to describe a propagating Gaussian-beam and hence show that the spot-size $w(z)$ a distance z from the beam-waist is given by

$$w^2(z) = w_0^2\left(1 + z^2 \Big/ z_R^2\right)$$

State the relationship between w_0 and z_R and describe their physical significance. [6]

Show that in the far-field, the full-angle divergence of a Gaussian-beam of wavelength λ and waist w_0 is given by

$$\theta = \frac{2\lambda}{\pi w_0} \qquad [2]$$

State the ABCD-law of Gaussian-beams [2]

A Gaussian-beam of waist w_1 is focused by a thin, positive lens of focal length f placed a distance z_1 beyond the waist, to form a new waist w_2 a distance z_2 beyond the lens.
Show that if $w_1 >> |z_1 - f|$ then

$$w_2 = \frac{f\lambda}{\pi w_1} \qquad [8]$$

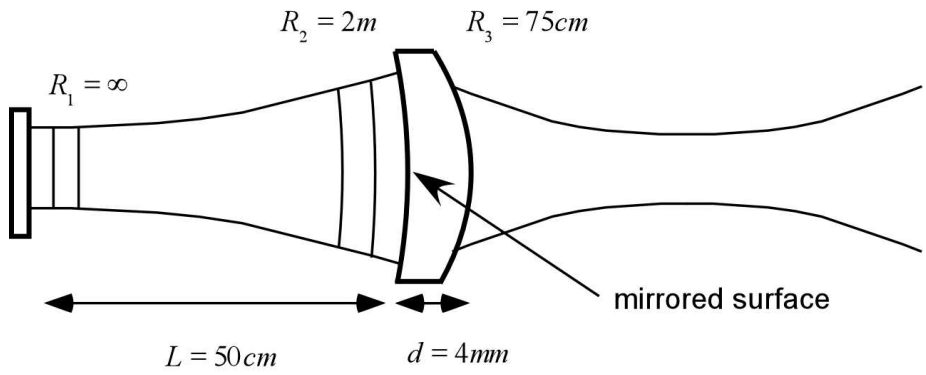
{ You may assume the following ray transfer matrices:

for translation through a distance d : $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

for a thin-lens of focal length f: $\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$ }

Question 2.

A helium-neon laser cavity consists, as shown in the figure, of a completely reflecting plane mirror placed a distance 50cm from the output mirror. The output mirror consists of a positive lens of thickness 4mm, made of glass of refractive index 1.5 with its inner surface coated to give a high reflectivity. The inner (reflecting) surface of the lens has radius of curvature 2m and outer surface a radius of curvature 75cm (both concave with respect to the laser beam propagation direction).



Find the spot-size of the laser beam at the plane mirror [4]

Find the ray-transfer matrix for the output coupler-lens [4]

Show, by calculating the ray transfer matrix neglecting the term describing propagation through the lens, that the output coupler is to a good approximation a *thin lens* and hence determine its focal length. [4]

Find the location of the beam waist outside the laser cavity. [4]

State the criterion generally used to determine the distance over which a laser beam is *collimated* and determine the distance over which the beam is collimated in this case ? [4]

{ You may assume the following ray transfer matrices:

for translation through a distance d : $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

for refraction at an interface of radius of curvature R from a dielectric medium of refractive index n to a medium of refractive index n' : $\begin{pmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n}{n'}-1\right) & \frac{n}{n'} \end{pmatrix}$

for a thin-lens of focal length f: $\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$ }