3C43 LASERS AND MODERN OPTICS

Problem sheet 2 – Lasers

(Answers to be handed in on Tuesday, 5th February 2002)

Question 1.

a. Describe, with reference to suitable energy-level diagrams, how a population inversion is achieved in

   i. a helium-neon laser, [4]
   ii. an ammonium maser. [4]

b. A helium-neon laser designed to produce 1mW of optical power at a wavelength of 632nm has a cavity length of 30 cm, which is formed of one totally reflecting and one partially reflecting mirror. The active medium, which can be assumed to occupy the entire laser cavity has a gain coefficient of 2.5% m^{-1} whilst the intra-cavity losses are equivalent to a loss coefficient of 1% m^{-1}. The gain profile has a Doppler-broadened width at the threshold gain of 1 GHz.

   i. Find the frequency separation of the longitudinal cavity modes and the number of modes that will oscillate when laser action occurs. [3]

   ii. Explain how your answer to i. would be different if the laser transition were *homogeneously-broadened.* [4]

   ii. Find the maximum value of the transmission coefficient of the partially reflecting cavity mirror for lasing action to be possible. [5]

Question 2.

Define the Einstein coefficients A_{21}, B_{21} and B_{12} for radiative transitions between an upper level 2 and a lower level 1 of an atom. [3]

Show, explaining any assumptions made, that the coefficients satisfy

$$g_2 A_{21} = \frac{8\pi h \nu^3 g_1 B_{12}}{c^3}$$

$$g_2 B_{21} = g_1 B_{12}$$

where g_1 and g_2 are the degeneracies of the levels. [6]

A model atom has three levels with energies $E_3 > E_2 > E_1$ and $g_2 = g_3 = 1$. Levels 2 and 3 are populated by unspecified processes at rates per unit volume R_2 and R_3 respectively. Spontaneous emission occurs between levels 3 and 2 and between levels 2 and 1 only.

Write down suitable rate-equations governing the time evolution of the populations of the atomic enery levels and hence derive an expression for the steady-state value of $N_3 - N_2$. [5]

What condition must $N_3 - N_2$ satisfy for laser action to be possible on the transition between levels 3 and 2 ? [1]

Show that for laser action to be possible, the pumping rates must satisfy

$$\frac{R_3}{R_2} > \frac{A_{32}}{A_{21} - A_{32}} \quad [2]$$

where A_{ij} is the Einstein coefficient for the transition between levels i and j.

In addition to the pumping of levels 2 and 3, the atom is now subjected to radiation at the frequency $\nu_{32} \equiv (E_3 - E_2)/h$ with an energy density $\rho(\nu_{32}) = \beta \frac{A_{32}}{B_{32}}$. Find the resulting percentage change in the value of $N_3 - N_2$. [3]