3C43 LASERS & MODERN OPTICS

Problem sheet 2 – Lasers

Question 1.

a) Explain the significance of the Einstein coefficients A_{21}, B_{21} and B_{12} for radiative transitions between an upper level 2 and a lower level 1 of an atom and state the equations defining them. [3]

b) Show, explaining any assumptions made, that the coefficients satisfy

$$g_2 A_{21} = \frac{8\pi h \nu^3 g_1 B_{12}}{c^3} \quad \text{and} \quad g_2 B_{21} = g_1 B_{12}$$

where g_1 and g_2 are the degeneracies of the levels. [5]

c) What is meant by the *spectral irradiance* of a beam of light? [2]

d) A beam of white light propagates along the *x*-axis and passes through an absorbing medium composed of two-level atoms occupying the space between *x=0* and *x=L*. If the spectral irradiance of the beam at frequency ν and at position *x* is $I_\nu(x)$ and the atomic lineshape function of the atomic medium is $g(\nu)$, centred on ν_0, show that

$$\frac{dI_\nu}{dx} = -\alpha(\nu)\cdot I_\nu \quad \text{where} \quad \alpha(\nu) \equiv g(\nu)\left(\frac{g_2}{g_1}N_1 - N_2\right)B_{21}\frac{nh\nu}{c},$$

and *n* is the refractive index of the medium [5]

e) Find an expression for $I_\nu(L)$, the spectral irradiance at frequency ν a distance *L* into the medium, if the spectral irradiance at the entrance to the medium is $I_\nu(0)$. [3]

f) As a result of passing through the absorbing medium, the spectrum of the light-beam has a hole in it centred on frequency ν_0. Show that if the absorption is very small, that the shape of the hole is directly proportional to $g(\nu)$. [2]

Question 2.

The gain-medium for a 3-level laser is composed of a gas of atoms with levels of energies $E_3 > E_2 > E_1$ and $g_1 = g_2 = g_3 = 1$. Atoms are pumped from level 1 to level 3 by an unspecified process at a constant rate R_3. Spontaneous emission can occur only between levels 3 and 2 and between levels 2 and 1 with rate coefficients A_{32} and A_{21} respectively. The laser cavity is designed to support a laser-mode at a single frequency $\nu = (E_2 - E_1)/h$.

a) Write down suitable rate-equations governing the time evolution of the populations of the atomic energy levels. [5]

b) Show that in the steady-state, the population of level 3 is $N_3 = R_3 / A_{32}$ [1]

c) If the total number of atoms in the gain medium is a constant *N*, find an expression for the steady-state population-inversion $N^* \equiv N_2 - N_1$ on the laser transition. [4]

d) What is the absolute minimum value of *N** that must be achieved by the pumping process if laser action is to be possible (assuming a loss-less laser cavity)? If this value of *N** is obtained for a pumping rate $R_3^{\min}$. Show that [3]

$$R_3^{\min} = \frac{N}{2/A_{21} + 1/A_{32}}$$

e) Because of cavity-losses, a higher value of the population inversion than that found in d) is actually necessary for laser action to occur. If the actual value of the threshold population-inversion is N_{th}^*, which is attained at a pump rate R_3^{th}, show that [3]

$$R_3^{th} = \left(N_{th}^* / N + 1\right) R_3^{\min}$$

f) If $A_{32} >> A_{21}$ show that this leads to

$$R_3^{th} \approx \frac{A_{21}}{2} N_{th}^* + \frac{A_{21}}{2} N$$

[2]

g) How does this compare to the expression that would have been obtained for a 4-level laser? [2]