3C43 LASERS & MODERN OPTICS

Problem sheet 1 – Matrix Optics

Question 1.

a) Define the coordinates (y, α) used in the ray transfer matrix formalism for the analysis of optical systems. [2]

b) State the conditions under which a 2x2 ray transfer matrix can be employed to describe an optical system [3]

c) An optical system has a transfer matrix $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$. Describe, with the aid of suitable diagrams, the effect of the system if

i) $C = 0$ and $D = M_1$

ii) $B = 0$ and $A = M_2$ [6]

d) An optical system occupies the region of space $0 < z \leq z_0$. In this region, the ray transfer matrix is

$$\begin{pmatrix} a & z \\ 0 & -\frac{1}{3} \end{pmatrix}$$

i) If the refractive index in the regions $z \leq 0$ and $z \geq z_0$ is n = 1, what is the value of a? [2]

ii) By considering the ray transfer matrix in the region $z > z_0$, and using the value of a found in i), find the location and magnification of the image of an object placed at $z=0$. [4]

Question 2.

a) Derive the ray transfer-matrix for refraction at the spherical interface, of radius of curvature R, between two dielectric materials, when light passes from a material of refractive index n to a material of refractive index n'. [4]

b) Hence write down the ray transfer-matrix for refraction at a plane interface. [2]

c) Show that the ray transfer-matrix for refraction at a plane interface from a material of refractive index n to a material of refractive index n' followed by propagation in the material of refractive index n' through a distance x is:

$$\begin{pmatrix} 1 & \frac{n}{n'}x \\ 0 & \frac{n}{n'} \end{pmatrix}$$

[4]

d) A stack of N dielectric layers, of which the mth layer has refractive index n_m, is surrounded by air (which has refractive index n_0). Show that the ray transfer matrix for the dielectric stack is

$$\begin{pmatrix} 1 & \frac{n_0}{n_{eff}}x \\ 0 & 1 \end{pmatrix} \quad \text{where} \quad n_{eff} = \left(\frac{1}{n_1} + \frac{1}{n_2} + \ldots + \frac{1}{n_N}\right)^{-1}$$

[6]

e) If the mth layer has $\frac{1}{n_m} = 0.5 + 0.05 \times (m-1)$, $n_0=1$ and $x = 1mm$, find:

i) the value of $\frac{1}{n_{eff}}$ [3]

ii) to 2 significant figures, the angle and elevation of the ray leaving the dielectric stack that enters the stack on the optic-axis inclined at an angle of $2°$. [4]