Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of marks for each subsection of a question.

Question 1

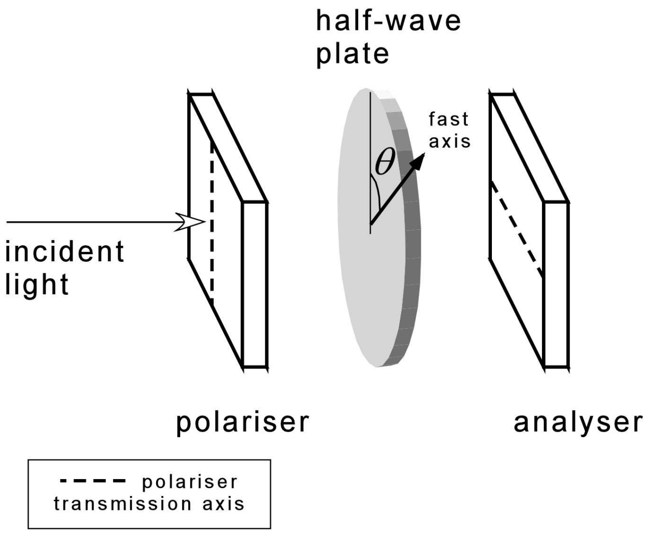
a) Show that if linearly polarised light is incident on a polariser such that the angle between the plane of polarisation of the light and the transmission axis of the polariser is θ, the fraction of the incident intensity that is transmitted is $\cos^2\theta$. [3]

b) If the light incident on the polariser in a) is unpolarised rather than linearly polarised, what fraction of the intensity is transmitted? [3]

c) What is meant by a *half-wave plate*? [4]

d) Show that if linearly polarised light is incident on a half-wave plate such that the plane of polarisation of the light makes an angle θ with the fast-axis of the wave-plate, the plane of polarisation of the light emerging from the wave-plate is rotated through an angle of magnitude 2θ. [5]

e) A half-wave plate is placed, as shown in the figure, between a crossed polariser and analyser such that the angle between the polariser transmission axis and the fast axis of the half-wave plate is θ.



If the light incident on the polariser from the left is unpolarised and has intensity I_0, show that the intensity of the light emerging to the right of the analyser is

$$I = \frac{I_0}{4}\left(1-\cos 4\theta\right).$$ [5]

(you may assume the identity $\cos 2x = 1 - 2\sin^2 x$).

Question 2

a) Show, detailing the approximations used, how the spherical-wave solution to the wave-equation can be modified to describe a propagating Gaussian-beam and hence show that the spot-size $w(z)$ a distance z from the beam-waist is given by

$$w^2(z) = w_0^2\left(1 + z^2/z_R^2\right).$$

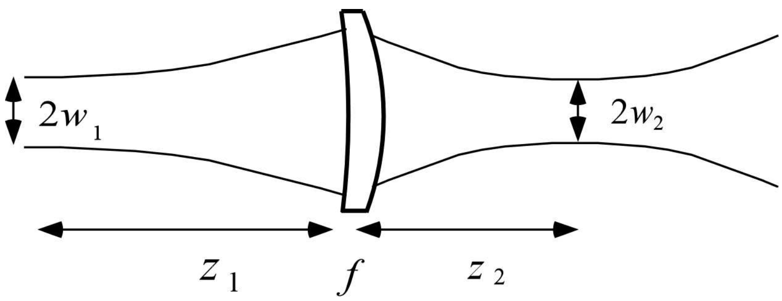
Show that $z_R = \dfrac{\pi w_0^2}{\lambda}$ where λ is the wavelength of the light and describe the physical significance of w_0 and z_R.

[10]

b) State the ABCD law of Gaussian-beams.

[2]

c) A Gaussian-beam of wavelength $\lambda = 628$ nm and waist $w_1 = 1\,\text{cm}$ is focused, as is shown in the figure, by a thin, positive lens of focal length $f = 5\,\text{cm}$ placed a distance $z_1 = 5.25\,\text{cm}$ beyond the waist, to form a new waist of spot size w_2 a distance z_2 beyond the lens.



By finding the ray transfer matrix describing the propagation of the beam between the waists and applying the ABCD law, determine w_2 and z_2.

[8]

{ You may assume the following ray transfer matrices:

for translation through a distance d : $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

for a thin-lens of focal length f: $\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$ }

Question 3

a) Describe, with reference to a suitable energy-level diagram, how a population inversion is achieved in a helium-neon laser. [6]

b) What is meant by the *brightness* of a light source? [2]

c) A helium-neon laser designed to produce 1mW of optical power at a wavelength of 632 nm has a cavity length of 30 cm. The active medium, which can be assumed to occupy the entire laser cavity has a gain coefficient of 2.5% m^{-1} whilst the intra-cavity losses are equivalent to a loss coefficient of 1% m^{-1}. The gain profile has a Doppler-broadened width, at the threshold gain, of 1 GHz and the intra-cavity waist size is 0.5mm.

   i. Find the frequency separation of the longitudinal cavity modes and hence the number of modes that will oscillate when laser action occurs. [3]

   ii. Explain, giving reasons, how your answer to i. would differ if the laser transition were *homogeneously broadened.* [3]

   iii. If one cavity mirror has a reflectivity of 100%, find the maximum value of the transmission coefficient of the other cavity mirror for lasing action to be possible. [3]

   iv. Calculate the brightness of the helium-neon laser, assuming that the full-angle divergence of the beam is given by $\theta = \frac{2\lambda}{\pi w_0}$ where w_0 is the intra-cavity waist size. [3]

Question 4

a) State the conditions under which a 2x2 ray transfer matrix can be used to describe an optical system. [2]

b) An optical system has a transfer matrix $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$. Describe, with the aid of suitable diagrams, the effect of the system if

i. B = 0
ii. C = 0. [5]

c) An optical system occupies the region of space $0 < z \leq z_0$. In this region, the ray transfer matrix is

$$\begin{pmatrix} -2 & z/2 \\ 0 & a \end{pmatrix}$$

i. Show that if the refractive index in the regions $z \leq 0$ and $z \geq z_0$ is n = 1 then $a = -\frac{1}{2}$. [2]

ii. By considering the ray transfer matrix in the region $z > z_0$, find the location and magnification of the image of an object placed at z=0. [5]

d) Show that the ray transfer matrix of a thin lens of focal-length f is

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}.$$ [3]

Hence show that the focal-length of a stack of N thin lenses, each of focal-length f, is f/N. [3]

{ You may assume that the ray transfer matrix for translation through a distance d is

$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$ }

Question 5

a) Define the *Einstein coefficients* A_{21}, B_{21} and B_{12} for radiative transitions between an upper level 2 and a lower level 1 of an atom.

Show that the B-coefficients satisfy $g_2 B_{21} = g_1 B_{12}$ where g_1 and g_2 are the degeneracies of the levels. [7]

b) A model atom has three levels with energies $E_3 > E_2 > E_1$ and $g_2 = g_3 = 1$. Levels 2 and 3 are populated by unspecified processes at rates per unit volume R_2 and R_3 respectively. Spontaneous emission occurs between levels 3 and 2 and between levels 2 and 1 only.

i. Write down suitable rate-equations governing the time evolution of the populations of the atomic energy levels and hence derive an expression for the steady-state value of $N_3 - N_2$ (you may assume that stimulated emission and absorption on the transition between levels 1 and 2 can be neglected). [8]

ii. What condition must $N_3 - N_2$ satisfy for laser action to be possible on the transition between levels 3 and 2 ? [1]

iii. Hence show that for laser action to be possible, the pumping rates must satisfy

$$\frac{R_3}{R_2} > \frac{A_{32}}{A_{21} - A_{32}}.$$ [2]

iv. In addition to the pumping of levels 2 and 3, the atom is now subjected to radiation at a frequency $\nu_{32} \equiv (E_3 - E_2)/h$ with an energy density $\rho(\nu_{32}) = 0.25 \frac{A_{32}}{B_{32}}$. Find the percentage change in the population difference $N_3 - N_2$ that results. [2]