

Mathematical Techniques 3

Green's Functions, PDEs and the Calculus of Variations

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Read this! I

These are not lecture notes!

These slides are merely an outline/list of what we will cover in the lectures. Use them as a guide, solve the problems indicated here, and follow-up on reading the material highlighted in the reference texts.

Outline of the Talk

- 1 Green's Functions
- 2 PDEs
- 3 The Calculus of Variations
- 4 Complex Analysis

Green's Functions I

We have used three sources for this topic:

- Ch. 15 from Riley, Hobson & Bence,
- Ch. 10 from Arfken, Weber & Harris (7th ed.),
- as well as the 2016 lecture notes.

Green's Functions II

Some parts of this topic are covered well in **AWH §10.1**, but Arfken *et al.* complicate things perhaps a bit too much. In the lectures we began with material from Arfken, but then moved on to use **RHB §15.2.5** which presented the material more clearly. Pay attention to the examples and exercises in Riley's book.

Pay attention to the various kinds of boundary conditions. These may be specified at both ends of the range, or as initial value conditions. Both are important.

Green's Functions III

We had spent some time on the Dirac δ -function as a prelude to Green's Functions. This is described in the 2016 lecture notes and in **AWH §1.11** and in **RHB §13.1.3**. This is an important topic as it is used in pretty much every brach of Physics, and, in this course, it is key to the method of Green's Functions.

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PDEs I

We have used two sources for this topic:

- Mainly the 2016 lecture notes,
- Most of Ch. 9 from Arfken, Weber & Harris (7th ed.): sections 9.2 to 9.7.

PDEs II

The main ideas here are:

- Characteristics of PDEs. **AWH §9.1,9.2**
- Second-order PDEs. **AWH §9.3**
- Separation of variables. **AWH §9.4**. We did not cover circular cylindrical coords. You have seen spherical polar coordinates in your Quantum modules.
- Laplace and Poisson and Wave equations. See 2016 lecture notes and **AWH §9.6**
- Diffusion equation. **AWH §9.7**

We did not cover Green's functions for PDEs. This topic is in the 2016 lecture notes, but was skipped.

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Calculus of Variations I

We have used three sources for this topic:

- The 2016 lecture notes,
- **RHB §22**
- **AWH §22** (yes, both books cover this topic in Ch 22!)

Calculus of Variations II

We covered material in these sections in the two books:

- **AWH §22.1**: This is a large section with many good problems. You should try to work through some of these as part of your revision.
- **AWH §22.2**: We covered part of this only. Topics were Hamilton's Principle which led to Noether's theorem and the conservation laws. See examples 22.2.1 and 22.2.2. We did not cover Hamilton's Equations.
- **RHB §22.1, 22.2, 22.5**: Here too you will find many excellent problems.

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Complex Analysis I

We did not cover anything from this topic. If you will be taking MT4, please read the 2016 lecture notes as you will benefit from some preparation of this awesome topic.

For more reading see [RHB §24](#) and [AWH §11](#).