Exercise 4

Physical constants

$M_\odot = 2 \times 10^{30}$ kg $R_\odot = 7 \times 10^{8}$ m $M_{\rm Sun} = 4.63$ (absolute magnitude) $L_\odot = 3.83 \times 10^{26}$ J s^{-1}
1 AU $= 1.5 \times 10^{11}$ m $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} $\sigma = 5.7 \times 10^{-8}$ kg s^{-3} K^{-4} (S-B constant)
$k_{\rm B} = 1.38 \times 10^{-23}$ m^2 kg s^{-2} K^{-1} $m_{\rm H} = 1.67 \times 10^{-27}$ kg

Assessed questions

Question 1
Consider a group of stars. Each star in the group has the same chemical composition, is homogeneous, and is composed of an ideal gas. Energy generation occurs through the P-P chain, with $\epsilon = \epsilon_{\rm pp}\rho T^4$ and $\epsilon_{\rm pp}$ being a constant. All of the energy is transported by radiation (and hence the temperature gradient is given by the expression derived in lectures for radiative zones) and the opacity is given by $\kappa = \kappa_0 \rho T^{-3.5}$, where κ_0 is a constant.
(i) Show that

$$M \propto R^{13}.$$

(ii) Also show that

$$L \propto M^{71/13}.$$

(iii) Obtain the slope of the line in an H-R diagram ($\log_{10} L$ versus $\log_{10} T_{\rm eff}$) that these stars lie on.

Question 2
The Schwarzschild condition for convective instability is

$$\frac{d\ln T}{d\ln P} > \frac{\gamma - 1}{\gamma}.$$

Consider a stellar atmosphere that is in radiative equilibrium, and which has a temperature profile

$$T^4 = \frac{3}{4} T_{\rm eff}^4 \left(\tau + \frac{2}{3}\right),$$

and pressure profile

$$P^2 = P_0^2 \ln\left(1 + \frac{3}{2}\tau\right),$$

where τ is the optical depth, and P_0 is a constant. Given that $\gamma = 5/3$, show that convection sets in where

$$\tau = \frac{2}{3}\left[\exp\left(\frac{4}{5}\right) - 1\right].$$

Non-assessed questions
(i) Make sure that you are able to derive the Schwarzschild criterion for convective instability.
(ii) Make sure that you understand which parameters control the radiative temperature gradient, and hence what determines whether or not a star has a convection zone or not.
(iii) For radiative models of stars, make sure you understand how scaling relations between mass, radius, temperature, luminosity can be obtained for specified opacity and energy generation rates.