Exercise 2

Physical constants

$M_\odot = 2\times10^{30}$ kg $R_\odot = 7\times10^{8}$ m $M_{Sun} = 4.63$ (absolute magnitude) $L_\odot = 3.83\times10^{26}$ J s^{-1}
1 AU $=1.5\times10^{11}$ m $G = 6.67\times10^{-11}$ N m^2 kg^{-2} $\sigma = 5.7\times10^{-8}$ kg s^{-3} K^{-4} (S-B constant)
$k_B = 1.38\times10^{-23}$ m^2 kg s^{-2} K^{-1} $m_H = 1.67\times10^{-27}$ kg

Assessed questions

Question 1

(i) Consider a plasma composed of fully ionised hydrogen and helium only. From first principles, show that the mean molecular weight is given by

$$\mu = \frac{4}{(3+5X).}$$

(ii) Calculate the value of μ if this gas has a helium mass fraction of 30%.
(iii) Consider an evolved star containing equal masses of fully ionised hydrogen and ^{12}C only. Calculate the value of μ for this stellar matter.

Question 2

Consider a uniform density star, composed of an ideal gas, that is in hydrostatic equilibrium. The density is given by $\rho = 3M/(4\pi R^3)$, where M is the stellar mass and R is the stellar radius.
(i) Obtain an expression for the gravitational acceleration at some arbitrary radius r inside the star.
(ii) Show that the central pressure is given by

$$P_c = \frac{3GM^2}{8\pi R^4}.$$

(iii) Show that the central temperature is given by

$$T_c = \frac{GM}{2R}\frac{\mu m_H}{k_B}.$$

(iv) For a star of Solar mass and radius, composed of 30% helium and 70% hydrogen by mass, where both constituents are fully ionised, calculate the central temperature and pressure.

Non-assessed questions

(i) Make sure that you are able to follow the derivation of the mean molecular weight and the equation of hydrostatic equilibrium in detail, as given in the online lecture notes.
(ii) Make sure also that you are able to follow and reproduce the derivation of the virial theorem, and be able to demonstrate that for a star in hydrostatic equilibrium without internal sources of energy such as fusion reactions, half of the change in gravitational potential energy goes into increasing the internal energy, and the other half is lost through radiation.