Exercise 1

Physical constants

$M_\odot = 2 \times 10^{30}$ kg $R_\odot = 7 \times 10^8$ m $M_{\rm Sun} = 4.63$ (absolute magnitude) $L_\odot = 3.83 \times 10^{26}$ J s^{-1}
1 AU $= 1.5 \times 10^{11}$ m $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} $\sigma = 5.7 \times 10^{-8}$ kg s^{-3} K^{-4} (S-B constant)
$k_{\rm B} = 1.38 \times 10^{-23}$ m^2 kg s^{-2} K^{-1} $m_{\rm H} = 1.67 \times 10^{-27}$ kg

Assessed questions

Question 1

(i) An advanced civilisation living on the planet Proxima b, which orbits at a distance of 0.05 AU from its host star Proxima Centauri, is able to measure parallax angles to a precision of 0.01". What is the greatest stellar distance that they are able to measure using the parallax method? Express your answer in light years.
(ii) The relation between stellar mass and luminosity has been measured to be $L_* \propto M_*^{3.5}$. Proxima Centauri has a mass $M_* = 0.12$ M$_\odot$. Assuming that this star is able to burn all of its hydrogen to form helium, estimate its main sequence life time. How does this compare with the main sequence life time of a Solar type star?
(iii) Given that Proxima's radius has been measured to be $R_* = 0.15$ R$_\odot$, estimate its Kelvin-Helmholtz time scale.

Question 2

A star has an apparent magnitude 8.25 and is located at a distance 85 parsecs from the Sun.
(i) What is its absolute magnitude?
(ii) What is its luminosity?
(iii) The star's radius is 0.5 R$_\odot$. What is its effective temperature?

Question 3

Consider a molecular cloud core composed entirely of molecular hydrogen (mean molecular weight $\mu = 2$) which has a uniform density and a temperature $T = 10$ K. The number density of molecules $n = 10^{12}$ m^{-3}, and the radius of the cloud $R = 3 \times 10^{16}$ m.
(i) Determine whether or not the cloud will be unstable to gravitational collapse.
(ii) Calculate the free fall time of the cloud.

Non-assessed questions

(i) Make sure that you are able to follow the derivation of the Jeans mass and the free-fall time in detail, as given in the online lecture notes.
(ii) In the lecture of week 2 we stated that the distribution of particle velocities in an ideal gas follows the Maxwell distribution

$$f(v) = 4\pi \left(\frac{m}{2\pi k_{\rm B} T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_{\rm B}T}\right) v^2 \qquad (1)$$

where m is the mass of the particle and $k_{\rm B}$ is Boltzmann's constant. Prove that

$$\int_0^\infty f(v)dv = 1,$$

and prove that

$$\left\langle \frac{mv^2}{2} \right\rangle = \frac{\int_0^\infty \frac{mv^2}{2} f(v)dv}{\int_0^\infty f(v)dv} = \frac{3}{2} k_{\rm B} T \qquad (2)$$

where $\langle X \rangle$ denotes the mean value of X. You may assume that

$$\int_{-\infty}^{\infty} \exp(-x^2)dx = \sqrt{\pi}.$$

Note that you will need to use integration by parts.