MATH6502 Example Sheet 7. Hand in all questions from section A.
Cover sheet with DEPARTMENT/TUTOR/YOUR NAME & signed.
Due into Maths room 6.10 by 2pm on Wednesday 10 December.

Section A

1. (a) Solve the following systems of simultaneous equations using standard Gaussian elimination. *Marks will be awarded for row sum checks.*

$$\text{(i)}\ \begin{aligned} x-2y&=3\\ 2x+y&=4\end{aligned} \qquad \text{(ii)}\ \begin{aligned} x_1+x_2+x_3&=6\\ x_1+2x_2+3x_3&=14\\ x_1+4x_2+9x_3&=36\end{aligned} \qquad \text{(iii)}\ \begin{aligned} x+y+z&=6\\ y&=2x\\ z&=x+y\end{aligned}$$

$$\text{(iv)}\ \begin{aligned} 2x+4y+7z&=25\\ 4x+5y+6z&=19\\ 3x+y-2z&=-10\end{aligned} \qquad \text{(v)}\ \begin{aligned} x+y-z&=-3\\ 2x+y+z&=4\\ 5x-y+2z&=23\end{aligned}$$

   (b) In each case of part (a), check that your solution satisfies all of the equations

   (c) In each case evaluate the determinant of the matrix of coefficients which defines the system.

2. Solve the following systems of equations using standard Gaussian elimination:

$$\text{(i)}\ \begin{aligned} 2x+3y&=5\\ 6x+9y&=15\end{aligned} \qquad \text{(ii)}\ \begin{aligned} 2x+3y&=1\\ 6x+9y&=5\end{aligned} \qquad \text{(iii)}\ \begin{aligned} -2x+y+z&=1\\ x-2y+z&=2\\ x+y-2z&=3\end{aligned} \qquad \text{(iv)}\ \begin{aligned} -2x+y+z&=1\\ x-2y+z&=2\\ x+y-2z&=-3\end{aligned}$$

Section B

1. Solve the following systems of simultaneous equations by backward substitution:

$$\text{(a)}\ \begin{aligned} 2x+3y-z&=12\\ 9y+z&=34\\ 4z&=28\end{aligned} \qquad \text{(b)}\ \begin{aligned} 5x-y+3z&=11.1\\ 4y-7z&=1.1\\ 3.1z&=2.17\end{aligned} \qquad \text{(c)}\ \begin{aligned} w+3x-y+z&=5\\ 2x+y+z&=7\\ y+2z&=6\\ 3z&=9\end{aligned}$$

2. Solve the following systems of equations:

$$\text{(i)}\ \begin{aligned} x+y&=3\\ x-y&=1\end{aligned} \qquad \text{(ii)}\ \begin{aligned} x+y&=3\\ x-y&=1.0001\end{aligned} \qquad \text{(iii)}\ \begin{aligned} x+y&=2\\ x+1.0001y&=2\end{aligned} \qquad \text{(iv)}\ \begin{aligned} x+y&=2\\ x+1.0001y&=2.0001\end{aligned}$$

   Comment: compare (i) and (ii) with (iii) and (iv). What are the corresponding determinants?

3. Solve the following systems of simultaneous equations using standard Gaussian elimination:

$$\text{(a)}\ \begin{aligned} x_1+x_2+x_3+x_4&=7\\ x_1+x_2+2x_4&=8\\ 2x_1+2x_2+3x_3&=10\\ -x_1-x_2-2x_3+2x_4&=0\end{aligned} \qquad \text{(b)}\ \begin{aligned} x_1+x_2+x_3+x_4&=7\\ x_1+x_2+2x_4&=5\\ 2x_1+2x_2+3x_3&=10\\ -x_1-x_2-2x_3+2x_4&=0\end{aligned}$$

$$\text{(c)}\ \begin{aligned} x_1+x_2+x_3&=6\\ x_1+2x_2+2x_3&=11\\ x_1+3x_2+2x_3&=13\end{aligned} \qquad \text{(d)}\ \begin{aligned} 2x_1+x_2+x_3&=8\\ x_1+2x_2+2x_3&=17\\ x_1+2x_2+3x_3&=17\end{aligned} \qquad \text{(e)}\ \begin{aligned} 6x_1+2x_2-3x_3&=5\\ -x_1+8x_2+3x_3&=-10\\ x_1+4x_2+12x_3&=12\end{aligned}$$

$$\text{(f)}\ \begin{aligned} x-y+5z&=-6\\ 3x+3y-z&=10\\ x+3y+2z&=5\end{aligned} \qquad \text{(g)}\ \begin{aligned} 2x+y+2z&=11\\ 2x-y-z&=-3\\ 3x+2y+z&=9\end{aligned} \qquad \text{(h)}\ \begin{aligned} 3x+2y+z&=9\\ 2x-y-z&=-3\\ 2x+y+2z&=11\end{aligned}$$

$$\text{(i)}\ \begin{aligned} 5x+3y-z&=0\\ 2x-3y+z&=0\\ x+4y-2z&=0\end{aligned} \qquad \text{(j)}\ \begin{aligned} x+y-3z&=1\\ 2x-y+z&=9\\ 3x+y-4z&=8\end{aligned} \qquad \text{(k)}\ \begin{aligned} x+y-z&=6\\ 2x-y+z&=-9\\ x-2y+3z&=1\end{aligned}$$

$$\text{(l)}\ \begin{aligned} 2x+y-z&=4\\ 4x+2y-z&=1\\ -2x+3y+5z&=0\end{aligned} \qquad \text{(m)}\ \begin{aligned} w+2x+3y-z&=6\\ 2w+4x-5y-2z&=1\\ 3w+x&=11\\ -w-x+y+z&=0\end{aligned} \qquad \text{(n)}\ \begin{aligned} w+2x-4y+z&=9\\ -w-2y+z&=1\\ w+x-y+z&=3\\ -2w-2x+2y-z&=-7\end{aligned}$$