MATH6502 Example Sheet 6. **Hand in all questions from section A.**

Cover sheet with DEPARTMENT/TUTOR/YOUR NAME & signed.
Due into Maths room 6.10 by 2pm on Wednesday 26 November.

Section A

1. Which of the following square matrices are symmetric?

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \quad \underline{\underline{B}} = \begin{pmatrix} 3 & 4 & 0 \\ 4 & 1 & -3 \\ 0 & -3 & 6 \end{pmatrix} \quad \underline{\underline{C}} = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix} \quad \underline{\underline{D}} = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}.$$

2. Find the determinants of the following 2×2 matrices and (where there is a relationship between two matrices) comment on how the matrices and their determinants relate to one another.

$$\text{(i)} \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} \quad \text{(ii)} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \text{(iii)} \begin{pmatrix} 3.1 & 2.6 \\ -1.1 & 4.2 \end{pmatrix} \quad \text{(iv)} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\text{(v)} \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix} \quad \text{(vi)} \begin{pmatrix} 1 & -2 \\ 3 & -8 \end{pmatrix} \quad \text{(vii)} \begin{pmatrix} a & b \\ \lambda c & \lambda d \end{pmatrix} \quad \text{(viii)} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\text{(ix)} \begin{pmatrix} 1 & 2 \\ 1 & 4 \end{pmatrix} \quad \text{(x)} \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix} \quad \text{(xi)} \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix} \quad \text{(xii)} \begin{pmatrix} 5 & 0 \\ 4 & 3 \end{pmatrix}.$$

3. Find the determinant of the matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$ and show that $\det(\underline{\underline{A}}\,\underline{\underline{B}}) = \det(\underline{\underline{A}})\det(\underline{\underline{B}})$.

$$\underline{\underline{A}} = \begin{pmatrix} 2 & -3 & -4 \\ 1 & 0 & -2 \\ 0 & -5 & -6 \end{pmatrix} \qquad \underline{\underline{B}} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 5 \\ 4 & 1 & 3 \end{pmatrix}.$$

Section B

1. Solve the following matrix multiplication for a, b, c and d: $\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

2. If $\underline{\underline{A}}\,\underline{\underline{B}} = \underline{\underline{A}}$ and $\underline{\underline{B}}\,\underline{\underline{A}} = \underline{\underline{B}}$ then show that $\underline{\underline{A}}^2 = \underline{\underline{A}}$.

3. Define D_n as the determinant of $\underline{\underline{A}}_n$ where $\underline{\underline{A}}_n$ are this series of $n \times n$ matrices:

$$\underline{\underline{A}}_1 = \begin{pmatrix} a \end{pmatrix} \quad \underline{\underline{A}}_2 = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \quad \underline{\underline{A}}_3 = \begin{pmatrix} a & b & 0 \\ b & a & b \\ 0 & b & a \end{pmatrix} \quad \underline{\underline{A}}_4 = \begin{pmatrix} a & b & 0 & 0 \\ b & a & b & 0 \\ 0 & b & a & b \\ 0 & 0 & b & a \end{pmatrix} \quad \ldots$$

   (a) Find D_1 and D_2.
   (b) What are the cofactors C_{11} and C_{12} for $\underline{\underline{A}}_n$?
   (c) Using (b), and expanding on the first row of $\underline{\underline{A}}_n$, show that $D_n = aD_{n-1} - b^2 D_{n-2}$.
   (d) Use (c) and (a) to show that $D_5 = a(a^2 - b^2)(a^2 - 3b^2)$.

4. Find the determinant of the following matrices:

$$\text{(i)} \begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & -1 & 2 \end{pmatrix} \quad \text{(ii)} \begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & -1 \\ 2 & -1 & 2 \end{pmatrix} \quad \text{(iii)} \begin{pmatrix} 1 & 3 & -1 \\ -1 & 1 & 4 \\ 2 & 2 & 3 \end{pmatrix}$$

$$\text{(iv)} \begin{pmatrix} a & 2z & 9.6 \\ 0 & b & \sqrt{2} \\ 0 & 0 & c \end{pmatrix} \quad \text{(v)} \begin{pmatrix} 1 & 4 & 7 \\ -1 & 3 & -2 \\ 0 & 1 & 3 \end{pmatrix} \quad \text{(vi)} \begin{pmatrix} a & b & c \\ -1 & x & 0 \\ 0 & -1 & x \end{pmatrix}$$

$$\text{(vii)} \begin{pmatrix} x & -1 & 0 \\ 0 & x & -1 \\ c & b & a+x \end{pmatrix} \quad \text{(viii)} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & -1 & 7 \end{pmatrix} \quad \text{(ix)} \begin{pmatrix} 2 & -3 & -4 & 0 \\ 1 & 0 & -2 & 0 \\ 0 & -5 & -6 & 0 \\ 7 & 5 & 3 & x \end{pmatrix}.$$

5. Show that $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 2\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 2\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 2\theta \end{vmatrix} = 2(1+2\sin 2\theta)$.