MATH6502 Example Sheet 1. Hand in all questions from section A.
Cover sheet with DEPARTMENT/TUTOR/YOUR NAME & signed.
Due into Maths room 6.10 by 2pm on Wednesday 8 October.

Section A

Note: apart from question 1, this is entirely revision from MATH6501.

1. Find the first three terms of the Taylor series for

$$f(x) = (x+1)^5 + e^x$$

near the point $x = 0$.

2. Use integration by parts to evaluate the following integrals:

$$\text{(i)} \int xe^x \, \mathrm{d}x \qquad \text{(ii)} \int_0^\pi x^2 \sin(nx) \, \mathrm{d}x$$

[You will have to integrate twice for (ii).]

3. (a) Using radians as a measure of angle, plot $\cos(x)$ and $\sin(x)$.
   (b) If n is a positive integer, express each of the following as either 0 or a power of (-1):

$$\text{(i)} \cos(n\pi) \qquad \text{(ii)} \sin(n\pi) \qquad \text{(iii)} \cos((2n+1)\pi/2) \qquad \text{(iv)} \sin((2n+1)\pi/2).$$

Section B

1. A function $f(x)$ is defined as being *even* if $f(-x) = f(x)$ and *odd* if $f(-x) = -f(x)$. For products of two general functions, show that when considering function types:
   (a) *even* $\times$ *even* $=$ *even*
   (b) *odd* $\times$ *odd* $=$ *even*
   (c) *even* $\times$ *odd* $=$ *odd*

2. Evaluate the following integrals:
   (a) $\displaystyle \frac{1}{L}\int_{-L}^{L} e^x \cos\left(\frac{n\pi x}{L}\right) \mathrm{d}x$
   (b) $\displaystyle \frac{1}{L}\int_{-L}^{L} e^x \sin\left(\frac{n\pi x}{L}\right) \mathrm{d}x.$

   You can either integrate by parts twice or use complex numbers to do both integrals at once by setting $\cos x + i \sin x = e^{ix}$.