UNIVERSITY COLLEGE LONDON
DEPARTMENT OF PHYSICS AND ASTRONOMY

2B21 MATHEMATICAL METHODS IN PHYSICS AND ASTRONOMY

Problem Sheet M6 (2003–2004)

Solutions to be handed in on Tuesday 18 November 2003

1. A function $u(x, y)$ of two independent variables x and y satisfies the first order partial differential equation

$$x\,\frac{\partial u}{\partial x} - y\,\frac{\partial u}{\partial y} = u\,.$$

By first looking for a separable solution of the form
$u(x, y) = X(x) \times Y(y)$, find the general solution of the equation. **[8 marks]**

Determine the $u(x, y)$ which satisfies the boundary condition
$u = x + x^3$ when $y = x$. **[2 marks]**

2. The potential $V(r, \theta)$ in plane polar coordinates satisfies the equation

$$\nabla^2 V(r,\theta) = \frac{1}{r}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial}{\partial r}V(r,\theta)\right) + \frac{1}{r^2}\,\frac{\partial^2}{\partial\theta^2}V(r,\theta) = 0\,.$$

By searching for a solution in the separable form, $V(r, \theta) = R(r) \times \Theta(\theta)$ show that the general solution in the region $0 \leq \theta \leq 2\pi$ is

$$V(r,\theta) = A + B\,\ell n\,r + \sum_{n=1}^{\infty}\left(C_n r^n + \frac{D_n}{r^n}\right)\left(E_n \cos n\theta + F_n \sin n\theta\right)\,.$$ **[7 marks]**

If the potential on the ring $r = a$ is given by $V(a, \theta) = V_0 \cos\theta$, evaluate the potential in the regions $0 \leq r \leq a$ and $a \leq r < \infty$. **[3 marks]**