1B45 Mathematical Methods Problem Sheet 1 Solutions 2005/2006

1.

$$x^2+5x-6 = x^2+6x-x-6 = x(x-1)+6(x-1) = (x+6)(x-1) = 0\ .$$

Thus the roots are $x=-6$ and $x=1$. [1]

$$3x^2-7x = x(3x-7) = 0 \quad , \quad x=0 \ \text{ and } \ x=\frac{7}{3}\ .$$

[1]

$$4x^2-8x+4 = 4x^2-4x-4x+4 = 4x(x-1)-4(x-4) = (4x-4)(x-1) = 0$$

Thus both roots are $x=1$. [1]

$$\text{The expression } \ \frac{1}{(x+1)}+\frac{2}{(x+2)}=1 \quad \text{simplifies to } \ x^2=2\ .$$

[1]

Thus the roots are $x=\pm\sqrt{2}$.

2.

$$\text{We have } \ 2x^2-3x-5 = x^2-\frac{3}{2}x-\frac{5}{2} = (x-\frac{3}{4})^2-\frac{9}{16}-\frac{10}{4}=0\ .$$

[2]

$$\text{Thus } \ x=\frac{3\pm\sqrt{49}}{4}\ .$$

$$\text{Now } \ x^2-2x+a=(x-1)^2-1+a=0 \ \text{ and } \ x=1\pm\sqrt{1-a}\ .$$

[2]

.

3. From $2x^2-7x+4=0 \quad \alpha+\beta=7/2$ and $\alpha\beta=2$ and the values of

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{7}{2}\times\frac{1}{2}=\frac{7}{4} \ \text{ and } \ \frac{1}{\alpha\beta}=\frac{1}{2}\ .$$

Thus the required quadratic is $x^2-\frac{7}{4}x+\frac{1}{2}=0$ or $4x^2-7x+2=0$. [4]

4.

$$\text{Since } \ E=E_1+E_2 \ \text{ and } \ sin\frac{\phi}{2}=\frac{m}{2\sqrt{E_1E_2}}\ , 4E_1E_2=\frac{m^2}{sin^2\frac{\phi}{2}}, \ \text{ or } \ 4E_1(E-E_1)=\frac{m^2}{sin^2\frac{\phi}{2}}$$

$$\text{from which we obtain } \ E_1^2-EE_1+\frac{m^2}{4\ sin^2\frac{\phi}{2}}=0\ .$$

[3]

Completing the square we find

$$\left(E_1-\frac{E}{2}\right)^2-\frac{E^2}{4}+\frac{m^2}{4sin^2\frac{\phi}{2}}=0 \ \text{ whence } \ E_1=\frac{E}{2}\pm\frac{1}{2}\sqrt{E^2-\frac{m^2}{sin^2\frac{\phi}{2}}}\ .$$

[3]

From the square root term we find $sin\frac{\phi}{2}=\frac{m}{E}$. [2]

5.

Using the factor theorem three times $x^3 + 2x^2 - x - 2 = (x-1)(x+2)(x+1)$.

(Some students will do this in part by long division - full credit if they get the correct result.) [**2**]

For $x^3 - x^2 - x - 2$ the factor theorem gives $(x-2)$ as a factor .

[**2**]

Long division, which the students should do leads to $x^3 - x^2 - x - 2 = (x-2)(x^2+x+1)$.

$$x^4 - 1 = (x^2-1)(x^2+1) = (x+1)(x-1)(x^2+1) \ .$$

[**2**]

$$x^{\frac{3}{2}} + a^{\frac{3}{2}} = (x^{\frac{1}{2}})^3 + (a^{\frac{1}{2}})^3 = (\sqrt{x} + \sqrt{a})(x - \sqrt{x}\sqrt{a} + a) \ .$$

(The expansion for $x^3 + a^3$ has been done in the lectures.) [**4**]