1B45 Mathematical Methods Problem Class 5 2005/2006
Week starting Monday 28th. November

1 (a) In the lectures it was proved that

$$(\vec{A} \times \vec{B}) \times \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{A}(\vec{B} \cdot \vec{C}) \ .$$

Use the same method to prove that

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) \ .$$

(b) Prove Lagrange's identity

$$(\vec{A} \times \vec{B}).(\vec{C} \times \vec{D}) = (\vec{A} \cdot \vec{C})(\vec{B} \cdot \vec{D}) - (\vec{A} \cdot \vec{D})(\vec{B} \cdot \vec{C}) \ .$$

(Hint - remember that the triple **scalar** product remains unchanged under cyclical permutation of the vectors.)

(c) Prove that

$$\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0 \ .$$

2. The position of a particle is given by

$$\vec{r} = A(e^{\alpha t}\hat{i} + e^{-\alpha t}\hat{j}$$

where A and α are constants. Find the magnitude of the velocity at time t.

3. Repeat the derivation for the velocity and acceleration in polar coordinates, done in the lectures.

A bead moves outward with a constant speed u along the spoke of a wheel. It starts at the center at $t = 0$ and the angular position of the spoke is given by $\theta = \omega t$, where ω is constant. Find the velocity and acceleration of the bead.

4. Particles P_1 and P_2 move around concentric circles of radii a_1 and a_2 in the same sense and with angular velocities ω_1 and ω_2, all respectively. Show that the angular velocity of P_2 about P_1 is given by

$$\Omega = \frac{1}{2}(\omega_1 + \omega_2) + \frac{1}{2}(\omega_1 - \omega_2)\frac{a_1^2 - a_2^2}{r^2}$$

where $r = P_1P_2$.

If P_1,P_2 represent two planets, it may be shown that $\omega_1 = \mu^{\frac{1}{2}}/a_1^{\frac{3}{2}}$, $\omega_2 = \mu^{\frac{1}{2}}/a_2^{\frac{3}{2}}$, where μ is a constant for the solar system. Derive that, in this case, the motion of P_2, as observed from P_1, reverses its direction when the angle θ between the radii to the planets is given by

$$\cos\theta = \frac{\sqrt{a_1 a_2}}{a_1 + a_2 - \sqrt{a_1 a_2}} \ .$$