1B45 Mathematical Methods Problem Class 4 2005/2006
Week starting Monday 21st. November

1. (a) Find the angle between the vectors $\vec{A} = (2, 3, -1)$ and $\vec{B} = (2, -1, 2)$.

(b) Construct unit vectors parallel to $\vec{A}$ and $\vec{B}$ of the previous example.

(c) Calculate the projection P (or component) of $\vec{A}$ on to (along) $\vec{B}$. What is the projection Q of $\vec{B}$ on to $\vec{A}$?

(d) Show that the vectors $\vec{A} = (4, 1, -2)$ and $\vec{B} = (1, -2, 1)$ are orthogonal.

(e) Find $\vec{A} \times \vec{B}$ when $\vec{A} = (2, 3, -1)$ and $\vec{B} = (-1, 3, 3)$.

(f) Hence find the angle between $\vec{A}$ and $\vec{B}$ and check your result using the scalar product.

(g) Find the triple scalar product of the vectors $\vec{A} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{B} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{C} = \hat{i} + 4\hat{j} + 4\hat{k}$.

2. Hailstones, falling vertically in still air, are observed by someone running at a constant speed of 15km/hour to be falling at an angle of 10° to the vertical. Draw a velocity diagram and hence determine the speed of the hailstones.

A wind starts to blow towards the runner, still running at 15km/hour who then observes the hailstones to be falling at an angle of 30° to the vertical. Assuming that the vertical component of the velocity of the hailstones is unaltered by the wind, determine the speed of the wind.

3. A pilot is required to fly a plane due East from a point A to another point B, a distance D apart, and then return due West to A. The speed of the plane relative to the air is u, but throughout the flight a wind is blowing with constant velocity $\underline{v}$. Determine the time for the total journey if:

a) the wind is blowing from East to West;

b) the wind is blowing North to South.