1B45 Mathematical Methods Problem Class 3 2005/2006

Week starting Monday 14th. November

Maxima / minima - one variable.

1. A piece of wire 40 cm long is to be used to form the perimeter of a square and a circle so that the total area (of the circle plus the square) is maximum. If the circle is of radius r, show that the area of the circle plus the square is given by

$$A = \pi r^2 + (10 - \frac{1}{2}\pi r)^2 \ .$$

Determine the value of r at the stationary point and the corresponding area A . Is this a maximum or a minimum? Comment on your result.

Coordinate transformation and partial derivatives.

2.

The coordinates (x, y) and (u_1, u_2) are related by the following transformation equations

$$u_1^2 - u_2^2 = x \quad \text{and} \quad 2u_1u_2 = y \ .$$

Obtain

$$\left(\frac{\partial x}{\partial u_1}\right)_{u_2}, \left(\frac{\partial y}{\partial u_1}\right)_{u_2}, \left(\frac{\partial x}{\partial u_2}\right)_{u_1} \quad \text{and} \quad \left(\frac{\partial y}{\partial u_2}\right)_{u_1} \ .$$

Show that

$$\sqrt{\left(\frac{\partial x}{\partial u_1}\right)^2 + \left(\frac{\partial y}{\partial u_1}\right)^2} = 2\sqrt{(u_1^2 + u_2^2)} = 2(x^2 + y^2)^{\frac{1}{4}} \ .$$

Stationary points in a function of two variables.

3.

Determine the stationary points of the function given by

$$f(x, y) = x^3 e^{(-x^2 - y^2)} \ .$$

You should find three points. Determine the nature of these stationary points. (This function appears (mysteriously??) on page 27 of the notes!!)

Stationary points subject to constraints.

4. Find the volume of the largest rectangular parallelipiped (that is a box with edges parallel to the coordinate axes), that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \ .$$