1B45 Mathematical Methods Problem Class 1 2005/2006

Week starting Monday 24th. October

Algebraic Topics.

1. By completing the square obtain the formula for the roots of the quadratic equation $ax^2 + bx + c = 0$.

What is the condition that a quadratic equation can be factorised?

If α and β are the roots of the quadratic equation given at the beginning of the question, show without using the formula, that

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a} .$$

Check the following equations whether they are factorisable and solve appropriately by either factorising or completing the square.. (You may leave the solutions obtained by completing the square in terms of surds.)

(a)

$$2x^2 + x - 3 = 0 ,$$

(b)

$$2x^2 + 7x + 3 = 0 \text{ and}$$

(c)

$$2x^2 - 9x + 5 = 0 .$$

2.Express the following as partial fractions.

(a)

$$\frac{x^2 + 3}{x(x^2 + 2)} ,$$

(b)

$$\frac{3}{x(3x - 1)^2} .$$

Trigonometry.

3.

(a)Starting from a right angled triangle and Pythagoras's theorem prove that

$$cos^2\theta + sin^2\theta = 1 , \ 1 + tan^2\theta = sec^2\theta \text{ and } cot^2\theta + 1 = cosec^2\theta .$$

(b) Starting from

$$sin(A + B) = sinAcosB + cosAsinB \text{ and } cos(A + B) = cosAcosB - sinAsinB$$

show that

$$cos2A = cos^2A - sin^2A = 1 - 2sin^2A \ , \ sinA = \frac{2tan(\frac{A}{2})}{1 + tan^2(\frac{A}{2})} \text{ and } cosA = \frac{1 - tan^2\frac{A}{2}}{1 + tan^2\frac{A}{2}} .$$

(c)Solve for the following for values of θ between 0 and 2π

$$2cos^2\theta - sin\theta = 1 \ , \ cos2\theta + 3sin\theta = 2 \text{ and } sin\theta + 2cos\theta = 1.$$

Hint - each of the principal results, one in (a) and two in (b) is useful in solving one each of the above equations.

Describing mathematically a simple situation.

4. The maximum possible efficiency ϵ of a power plant is given by

$$\epsilon = 1 - \frac{T_C}{T_H}$$

where T_C is the temperature of the cooling water and T_H the maximum steam temperature.

Obtain a formula for the fractional change in efficiency $\frac{\Delta\epsilon}{\epsilon_1}$ where $\Delta\epsilon = \epsilon_2 - \epsilon_1$ if the temperature of the cooling water changes from T_{C1} to T_{C2} where $T_{C2} > T_{C1}$.

In the summer of 2003 the temperature of the cooling water of continental European power plants rose by $15K$. If T_H for a gas plant and for a PWR nuclear plant are $675K$ and $525K$ respectively calculate the fractional change in the maximum possible efficiencies for both type of power plant, given that T_{C1} is normally $290K$.

A much harder question both conceptually and technically.

5. A car is at the origin of a coordinate system at $t = 0$ and is travelling along the positive z axis with speed v so that after time t its x, y, z coordinates are $(0, 0, vt)$. The car emits a sharp pulse of sound which travels through still air with speed c_s. An observer at coordinates (x, y, z) detects the sound pulse at time t. Explain why the time at which the pulse was emitted is given by $[t]$ where

$$[t] = t - \frac{[|\vec{r}|]}{c_s} \quad \text{where} \quad [|\vec{r}|] = \sqrt{(z - v[t])^2 + x^2 + y^2)}$$

Obtain an expression for $[t]$.