1B45 Mathematical Methods Problem Sheet 10 2005/2006

Staple securely your answer sheets together and put **your name** and your **tutor's name** (Prof. T. W. Jones if you are not in the P+A department) on your script.

Please put your solutions in Prof. T. W.Jones's mail box by Friday 13th. January 2006.

1.

Identify the following surfaces, defined by the position vector $\vec{r}$ where k, l, n and m are fixed scalars, and $\hat{u}$ a fixed unit vector:

(a)$|\vec{r}| = k$;

(b)$\vec{r} \cdot \hat{u} = l$;

(c)$\vec{r} \cdot \hat{u} = m|\vec{r}|$ for $-1 \leq m \leq +1$ and **[4]**

(d)$|\vec{r} - (\vec{r} \cdot \hat{u})\hat{u}| = n$.

Find the angle between the position vectors to the points $(3, -4, 0)$ and $(-2, 1, 0)$ and find the direction cosines (ie the cosines of the angles a vector makes w. r. t. the $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ unit vectors), of a vector perpendicular to both position vectors. **[6]**

Use the vector product to determine the direction of the line of intersection of the two planes

$$x + 2y + 3z = 0 \text{ and } 3x + 2y + z = 0 .$$

Find the direction cosines of the line of intersection. **[3]**

Repeat the above by determining two points common to both planes. **[2]**

2.

(a)Evaluate $Re\,(e^{2iz})$ where $z = x + iy$. **[2]**

(b)Use the Argand diagram to show that $(-1 + \sqrt{3}i) = 2e^{\frac{2}{3}\pi i}$.

Then determine $(-1 + \sqrt{3}i)^{\frac{1}{2}}$. **[4]**

(c)Show that $\sqrt{i} = \frac{1+i}{\sqrt{2}}$ and $\frac{-1-i}{\sqrt{2}}$.

Then evaluate $|e^{\sqrt{i}}|$ recalling that $|z| = (zz^*)^{\frac{1}{2}}$. **[4]**

(d)Show that $i^i = e^{-\frac{\pi}{2}-2\pi n}$. **[2]**

(e)A is an oscillating physical quantity of the form $A = ReA_0e^{i\omega t}$. If the physical quantity B , $B = ReB_0e^{i\omega t}$ is predicted by theory to be related to A by a relationship of the form

$$B_0 = \sqrt{i}\ C\ A_0 \text{ where } C \text{ is a constant}$$

what can you conclude about the inter relationship of B and A? **[3]**