

1B45 Mathematical Methods Problem Sheet 9 2005/2006

Staple securely your answer sheets together and put **your name** and your **tutor's name** (Prof. T. W. Jones if you are not in the P+A department) on your script.

Please put your solutions in Prof. T. W. Jones's mail box by Friday 13th. January 2006.

1.

Show, by eliminating x and then y to get cartesian forms, that the line

$$2x + y + 3z - 1 = 0 = x + 10y - 21$$

can be written in vector form as $\vec{r}_1 = (1\hat{i} + 2\hat{j} - 1\hat{k}) + (-30\hat{i} + 3\hat{j} + 19\hat{k})\lambda_1$.

(This line is along the intersection of the two planes given in cartesian form at the beginning of the question. In the lectures we got such an intersection line using vector methods.)

Show also that the line

$$2x - y = 0 = 7x + z - 6$$

can be written in vector form as $\vec{r}_2 = (0\hat{i} + 0\hat{j} + 6\hat{k}) + (\hat{i} + 2\hat{j} - 7\hat{k})\lambda_2$.

[4]

Show that these lines intersect by determining the minimum distance between them. Also determine the coordinates of the intersection point (by using the cartesian forms of the lines), and the equation of the plane containing the lines.

[6]

2.

Find the foot of the perpendicular from the point $(1, 2, 1)$ for the line joining the origin to the point $(2, 2, 5)$.

[10]

(Hint - the foot of the perpendicular is perpendicular to the line.)

3.

Obtain the Maclaurin or Taylor series, as appropriate, for the following, writing down your result as well in the sigma notation.

$$e^x$$

$$\cos x$$

$$\sin x$$

$$\ln(1+x) \text{ and}$$

$$\ln \frac{1+x}{1-x}.$$

[10]