# 1B45 Mathematical Methods Problem Sheet 8 2005/2006

**Staple** securely your answer sheets together and put **your name** and your **tutor's name** (Prof. T. W. Jones if you are not in the P+A department) on your script.

Please hand in your solutions in the Monday lecture 5th December 2005.

**This problem sheet is one of the two you will be required to attain a mark of at least 24/30 before you can complete the course. You can have as many attempts as you need to attain this mark and the solutions will not be put on the web. A reasonable level of presentation is expected.**

1(a) If $\vec{A} = 3\hat{i} + 5\hat{j} - 7\hat{k}$ and $\vec{B} = 2\hat{i} + 7\hat{j} + \hat{k}$, find

$\vec{A}+\vec{B}$ , $\vec{A}-\vec{B}$ , $|\vec{A}|$ , $|\vec{B}|$ , $\vec{A}\cdot\vec{B}$ and the cosine of the angle between the vectors.

**[6]**

(b) Find the unit vector perpendicular to $\hat{i} + \hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + 3\hat{k}$. **[2]**

(c) If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ , $\vec{b} = -\hat{i} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$ obtain

$$\vec{a}\cdot\vec{b} \ , \ \vec{a}\times\vec{b} \ , \ \vec{a}\cdot(\vec{b}\times\vec{c}) \text{ and } \vec{a}\times(\vec{b}\times\vec{c}).$$

**[4]**

(d) Let $\vec{A}$ be an arbitrary vector and $\hat{n}$ a unit vector pointing in an arbitrary direction. Show that $\vec{A}$ may be expressed as

$$\vec{A} = (\vec{A}\cdot\hat{n})\hat{n} + (\hat{n}\times\vec{A})\times\hat{n} \ .$$

**[3]**

(e) Show that the position vector $\vec{r}$ of an aircraft circling above an airfield at height $h$ in a circle of radius $a$ with constant speed $V$ can be expressed as

$$\vec{r} = a\left(\hat{i}cos(Vt/a) + \hat{j}sin(Vt/a)\right) + h\hat{k}$$

in terms of the time $t$.

Find the velocity and acceleration vectors of the aircraft. **[6]**

(f) A particle is moving with constant velocity $\vec{v} = u\hat{i}$ along the line $y = 2$. Describe $\vec{v}$ in polar coordinates. **[3]**

(g) A bead moves along the spoke of a wheel at constant speed $u$ meters per second. The wheel rotates with uniform angular velocity $d\theta/dt = \omega$ about an axis perpendicular to the $xy$ plane. At $t = 0$ the spoke is along the $x$ axis, and the bead is at the origin. Find the velocity at time $t$

in polar coordinates,

in cartesian coordinates. **[6]**