1B45 Mathematical Methods Problem Sheet 6 2005/2006

Staple securely your answer sheets together and put **your name** and your **tutor's name** (Prof. T. W. Jones if you are not in the P+A department) on your script.

Please hand in your solutions in the Monday lecture 21 st. November 2005.

This problem sheet is one of the two you will be required to attain a mark of at least 24/30 before you can complete the course. You can have as many attempts as you need to attain this mark and the solutions will not be put on the web.

It contains standard bookwork material, differentiation and simple integration.

A reasonable level of presentation is expected.

1.(a) Write down the definition of the derivative $\frac{df(x)}{dx}$ in terms of a limiting procedure. [1]

(b) By long division show that [1]

$$x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + ... + a^{n-2}x + a^{n-1}) \quad (n \geq 2) \ .$$

(c) From the above obtain the standard result that [1]

$$\frac{d\ x^n}{dx} = nx^{(n-1)} \ .$$

(d)Using the above result (assumed valid for all n values), the product and chain rules as appropriate obtain the derivatives of the following
$x^8, \quad x^{\frac{1}{2}}, \quad x^{-3}, \quad x^{1.5}, \quad x^{-\frac{1}{2}}, \quad x^{-4}$ and $x^2(1-x)^{1/2}$. [4]

(e)

$$\text{If } \ x = \frac{\epsilon}{kT} \ \text{ and } \ C_v = Nk\frac{x^2e^x}{(1+e^x)^2} \ \text{ determine } \ \frac{dC_V}{dT} \ .$$

[3]

2. (a) Write down the definition of the definite integral in terms of a limiting procedure of elementary areas. [1]

(b) Show that that the derivative of an indefinite integral of $f(x)$ is $f(x)$. [1]

(c) Integrate $x^8, \quad x^{\frac{1}{2}}$, and x^{-3}. [3]

(d) Explain the method of 'integration by parts'. [1]

(e) Evaluate the following
$\int \ln\ x\ dx, \quad \int xe^{ax}dx, \quad \int x\ cosx\ dx$, and $\int x^2 sinx\ dx$. [4]

3. (a) Apply the definition in 1(a) to the function $y = a^x$, where a is a constant and explain carefully how the exponential function is defined by the following expression [2]
$y = \frac{dy}{dx} = e^x$.

(c) How do the above considerations in (b) allow one to determine $\int \frac{dy}{y}$? [1]

(d) Use simple methods to obtain the following integrals
$\int tan\theta d\theta, \quad \int \frac{1}{ax+b}dx, \quad \int \frac{dx}{x^2}$, and $\int \frac{dx}{(ax+b)^2}$. [4]

(e) Make the obvious substitution to obtain $\int \frac{dx}{\sqrt{x}+2}$. [3]

(**Note the denominator is $x^{1/2} + 2$!!**)