1B45 Mathematical Methods Problem Sheet 5 2005/2006

Staple securely your answer sheets together and put **your name** and your **tutor's name** (Prof. T. W. Jones if you are not in the P+A department) on your script.

Please put your solutions in Prof. T. W. Jones's mail box by Friday 11 th. November 2005.

1.

Starting from $sin(A+B) = sinAcosB+cosAsinB$ show that $sin\ x = \dfrac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}} = \dfrac{2t}{1+t^2}$.

$$\text{If } \ t = tan\frac{x}{2} \ \text{ show that } \ dx = \frac{2dt}{1+t^2} \ .$$

$$\text{Hence show that } \ \int cosec\ x\ dx = ln\ tan\frac{x}{2} \ .$$

[10]

2.

The equation of motion for a truck leaking sand is given by

$$\left(M + m(1 - \frac{t}{T})\right)\frac{dv}{dt} = F$$

where M,m,T and F are constants, v is the speed and $\frac{dv}{dt}$ is the acceleration of the truck.

If the truck's initial speed at $t = 0$ is zero and the final speed at $t = T$ is v_f show that **[10]**

$$v_f = \frac{FT}{m} ln\ \frac{M+m}{M} \ .$$

3.

Assuming that the volume under the surface $e^{-\alpha(x^2+y^2)}$ equals I^2

$$\text{where } \ I = \int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \int_{-\infty}^{\infty} e^{-\alpha y^2} dy \ \text{ show that}$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}} \ \text{ and } \ \int_0^{\infty} e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \ .$$

$$\text{Show that } \ \int_0^{\infty} x^2 e^{-\alpha x^2} dx = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}} \ .$$

The distribution of the speed v of molecules, mass m, in a gas in thermal equilibrium at temperature T is given by

$$P(v)dv = 4\pi N\ v^2 e^{-\frac{1}{2}\frac{mv^2}{kT}} dv$$

where k is the Boltzmann constant and N is a normalizing constant. **[10]**

Determine N such that $\int_0^{\infty} P(v)dv = 1$.