

1B45 Mathematical Methods Problem Sheet 4 2005/2006

Staple securely your answer sheets together and put **your name** and your **tutor's name** (Prof. T. W. Jones if you are not in the P+A department) on your script.

Please hand in your solutions at the Friday Lecture on 4th. November 2005

1. From the definitions of $\cosh z$ and $\sinh z$ show that

$$\cosh z + \sinh z = e^z$$

and

$$\cosh^2 z - \sinh^2 z = 1.$$

[3]

Show that if $y = \cosh^{-1} \frac{x}{a}$ then

$$y = \cosh^{-1} \frac{x}{a} = \ln \left[\frac{x \pm \sqrt{x^2 - a^2}}{a} \right] = \pm \ln \left[\frac{x + \sqrt{x^2 - a^2}}{a} \right]$$

giving full details of how you arrive at the last expression.

[5]

If $y = \sinh^{-1} \frac{x}{a}$ show that

$$y = \sinh^{-1} \frac{x}{a} = \ln \left[\frac{x + \sqrt{x^2 + a^2}}{a} \right]$$

explaining why only the positive square root is taken.

[4]

If $y = \tanh^{-1} \frac{x}{a}$ show that

$$y = \tanh^{-1} \frac{x}{a} = \frac{1}{2} \ln \frac{a+x}{a-x}$$

[3]

2. In the following use the product and/or the chain rules.

(a) Find the first derivative (d/dx) of $x^2 e^x$.

[3]

(b) Show that

$$\frac{d}{dx} \ln(a^x + a^{-x}) = \frac{(a^x - a^{-x})}{(a^x + a^{-x})} \ln a$$

[3]

(c) Find the first derivative of $\ln(x^a + x^{-a})$.

[3]

(d) Find the first derivative of x^x

[3]

(e) Find the first derivative with respect to r , (regarding θ as a constant) of

[3]

$$\frac{1}{(r^2 + d^2 - 2rd\cos\theta)^{\frac{1}{2}}}.$$

(Note that this derivative is a partial derivative - see later in the course.)