1B45 Mathematical Methods Problem Sheet 3 2004/2005

Staple securely your answer sheets together and put **your name** and your **tutor's name** (Prof. T. W. Jones if you are not in the P+A department) on your script.

Please hand in your solutions at the Friday Lecture on 29 th. October 2004

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1. Find the real root(s) of the equation

$$2^{2x} + 3(2^x) - 4 = 0$$

(Hint - make a substitution and first solve the 'hidden' quadratic). **[6]**

2. A problem from mechanics requiring algebraic and trigonometric manipulation.

In a certain collision problem the kinematics are determined by the following three equations.

$$u = v_1 cos\theta + \sqrt{2}v_2,$$

$$v_1 sin\theta = \sqrt{2}v_2 \quad \text{and}$$

$$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2$$

Use the first two equations to express v_1 and v_2 in terms of u, $sin\theta$ and $cos\theta$.

Then substitute in the third equation and obtain solutions for $sin\theta$. **[12]**

3. A problem invoving algebraic manipulation particularly the manipulation of indices (or powers).

The flow of gas in a nozzle is given by

$$|\vec{v_2}| = \left[2\frac{\gamma}{(\gamma-1)}\left(p_1\nu_1 - p_2\nu_2\right)\right]^{\frac{1}{2}}$$

where $|\vec{v_2}|$ is the speed of the gas when the pressure and volume per unit mass are p_2 and ν_2 respectively. At the entrance of the nozzle the gas speed is much less than $|\vec{v_2}|$ and the pressure and volume per unit mass there are respectively p_1 and ν_1.**You dont need to know any of this to do this problem!!**

Using $p_1\nu_1^\gamma = p_2\nu_2^\gamma$ show that

$$|\vec{v_2}| = \left[2\left(\frac{\gamma}{\gamma-1}\right)p_1\nu_1\left(1-\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}\right)\right]^{\frac{1}{2}} \quad \text{and}$$

$$\frac{|\vec{v_2}|}{\nu_2} = \left[\frac{2p_1}{\nu_1}\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}}\left(1-\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}\right)\right]^{\frac{1}{2}}$$

[12]