UNIVERSITY OF LONDON

MSc/MSci EXAMINATION 2016

For Students of the
University of London

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH4211: STATISTICAL MECHANICS
PH5911: STATISTICAL MECHANICS
PH5911R: STATISTICAL MECHANICS – PAPER FOR RESIT CANDIDATES

Time Allowed: **TWO AND A HALF hours**

Answer **THREE** questions
No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

The total available marks add up to 120

All College-approved Calculators are permitted

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	$=$	4π	$\times$	10^{-7}	H m^{-1}
Permittivity of vacuum	ε_0	$=$	8.85	$\times$	10^{-12}	F m^{-1}
	$1/4\pi\varepsilon_0$	$=$	9.0	$\times$	10^{9}	m F^{-1}
Speed of light in vacuum	c	$=$	3.00	$\times$	10^{8}	m s^{-1}
Elementary charge	e	$=$	1.60	$\times$	10^{-19}	C
Electron (rest) mass	m_e	$=$	9.11	$\times$	10^{-31}	kg
Unified atomic mass constant	m_u	$=$	1.66	$\times$	10^{-27}	kg
Proton rest mass	m_p	$=$	1.67	$\times$	10^{-27}	kg
Neutron rest mass	m_n	$=$	1.67	$\times$	10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	$=$	1.76	$\times$	10^{11}	C kg^{-1}
Planck constant	h	$=$	6.63	$\times$	10^{-34}	J s
	$\hbar = h/2\pi$	$=$	1.05	$\times$	10^{-34}	J s
Boltzmann constant	k	$=$	1.38	$\times$	10^{-23}	J K^{-1}
Stefan-Boltzmann constant	σ	$=$	5.67	$\times$	10^{-8}	W m^{-2} K^{-4}
Gas constant	R	$=$	8.31			J mol^{-1} K^{-1}
Avogadro constant	N_A	$=$	6.02	$\times$	10^{23}	mol $^{-1}$
Gravitational constant	G	$=$	6.67	$\times$	10^{-11}	N m^{2} kg^{-2}
Acceleration due to gravity	g	$=$	9.81			m s^{-2}
Volume of one mole of an ideal gas at STP		$=$	2.24	$\times$	10^{-2}	m^{3}
One standard atmosphere	P_0	$=$	1.01	$\times$	10^{5}	N m^{-2}

MATHEMATICAL CONSTANTS

$$e \cong 2.718 \qquad \pi \cong 3.142 \qquad \log_e 10 \cong 2.303$$

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1. (a) The partition function Z for a system of N indistinguishable non-interacting particles may be approximated by

$$Z = \frac{1}{N!} z^N$$

where z is the partition function for a single particle.
Explain the arguments that lead to this approximation. [6]

(b) The Helmholtz free energy $F = E - TS$ is given by

$$F = -kT \ln Z$$

where the symbols have their usual meanings. Show that the pressure p is given by

$$p = kT \left. \frac{\partial \ln Z}{\partial V} \right|_{T,N} .$$

[6]

(c) The partition function for a single particle moving freely in a volume V is given by

$$z = V \left(\frac{mkT}{2\pi\hbar^2} \right)^{3/2} .$$

By evaluating the pressure for an assembly of N such indistinguishable particles, show that this results in the equation of state for an ideal gas. [6]

(d) The quantity $\Lambda = \sqrt{2\pi\hbar^2/mkT}$ is known as the *thermal de Broglie wavelength*. Explain the physical interpretation of this. [6]

(e) In the van der Waals description of an interacting gas the single particle partition function is approximated by

$$z = \frac{V - V_{\mathrm{ex}}}{\Lambda^3} e^{-\langle E \rangle / kT} .$$

Discuss how the various features of the interparticle interaction are accounted for through the quantities V_{ex} and $\langle E \rangle$. [6]

(f) Why is this approach referred to as a *mean field* approximation? [4]

(g) Sketch a number of van der Waals $p - V$ isotherms and identify the stable, metastable, and unstable regions. [6]

2. (a) In the Weiss model of ferromagnetism it is assumed that the magnetic moments are subject to an additional 'mean' magnetic field

$$\mathbf{b} = \lambda \mathbf{M}$$

where $\mathbf{M}$ is the magnetization and λ is a constant.

Explain briefly the origin of this field. [8]

(b) The magnetization of a collection of N non-interacting spin $\frac{1}{2}$ magnetic moments μ is given by

$$M = M_0 \tanh\left(\frac{\mu B}{kT}\right)$$

where M_0 is the saturation magnetization $M_0 = N\mu$ and the directions of $\mathbf{M}$ and the applied magnetic field $\mathbf{B}$ are parallel.

Show that the Weiss model leads to a spontaneous magnetization, in the absence of an external magnetic field, given by

$$\frac{M}{M_0} = \tanh\left(\frac{M}{M_0}\frac{T_\mathrm{c}}{T}\right)$$

where $T_\mathrm{c} = \lambda M_0^2/Nk$. [8]

(c) i. Sketch the behaviour of the spontaneous magnetization as a function of temperature. [4]

ii. What is the interpretation of T_c? [2]

iii. Discuss the order of the transition. [2]

(d) When T is very close to T_c then M/M_0 is very small. By expanding the $\tanh$ ($\tanh x \approx x - x^3/3 + \ldots$) show that, in the vicinity of T_c, the magnetization behaves as

$$\frac{M}{M_0} \propto \left(1 - \frac{T}{T_\mathrm{c}}\right)^{1/2}.$$

[8]

(e) How does this result compare with the behaviour of real systems? Discuss the difference. [8]

3. (a) i. What is the *order parameter* in a phase transition? [2]

   ii. Sketch and describe the difference in the order parameter for a *first order* and a *second order* phase transition. [4]

(b) The ferroelectric transition is a *displacive transition*.

   i. What does this mean? [2]

   ii. What symmetry is broken in this transition? [2]

   iii. What is the order parameter for this transition? [2]

(c) When the Landau theory of phase transitions is applied to the ferroelectric transition the free energy is written as a polynomial of the form

$$F = F_0 + F_2\phi^2 + F_4\phi^4 + F_6\phi^6$$

where ϕ is the order parameter.

   i. Explain why the power series is terminated. [2]

   ii. Discuss why there are no odd powers of ϕ in the expansion. [2]

(d) Show, by the use of sketches, how the above polynomial for the free energy can lead to a first order transition. [6]

(e) Under what circumstance would the transition become second order? [4]

(f) When the transition is first order show that the discontinuity in the order parameter at the transition is given by

$$\Delta\phi = \sqrt{\frac{-F_4}{2F_6}}$$

and discuss what happens to this as the transition becomes second order. [8]

(g) The Landau theory is not well-suited to the treatment of first order transitions.

   i. Explain why this is. [3]

   ii. Discuss the conditions under which it might be a reasonable approximation. [3]

4. (a) Explain what is meant by *phase space* in the context of classical Statistical Mechanics and contrast the Boltzmann and the Gibbs conception of phase space. [6]

   (b) Sketch and contrast the trajectories of an undamped harmonic oscillator and a damped harmonic oscillator in phase space. [6]

   (c) Give an expression for the entropy of a classical system in terms of ρ, the density of points in phase space. [5]

   (d) Liouville's theorem states that as a system evolves in time the density of points in phase space remains constant. This is incompatible with the Second Law of thermodynamics. Explain clearly this contradiction. [7]

   (e) Outline the resolution of this paradox by the use of *coarse graining* and discuss how quantum mechanics may be invoked to justify the procedure. [8]

   (f) Quantum mechanics is also important in understanding the Third Law of thermodynamics. Explain why this is, and discuss how the Third Law would be stated in a purely classical, non-quantum, world. [8]

5. Albert Einstein wrote:

> A theory is the more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended is its area of applicability.
>
> Therefore the deep impression which classical thermodynamics made upon me; *it is the only physical theory of universal content concerning which I am convinced that, within the framework of the applicability of its basic concepts, will never be overthrown*.

And Lev Landau wrote:

> Statistical physics and thermodynamics together form a unit.
>
> All the concepts and quantities of thermodynamics follow most naturally, simply and rigorously from the concepts of statistical physics.
>
> Although the general statements of thermodynamics *can* be formulated non-statistically, their application to specific cases always requires the use of statistical physics.

Write an essay discussing these contrary views. You should:

- give examples supporting the Landau view, and examples supporting Einstein's view; [15]
- discuss the extent to which the work of both Landau and Einstein were actually more in line with the philosophy of the other; [15]
- include mention of *emergence* as a key concept that resolves the Landau-Einstein conflict. [10]

END