Queen Mary
University of London

SCHOOL OF MATHEMATICAL SCIENCES

UNDERGRADUATE HANDBOOK
PART 7: MODULE DETAILS

2009–10

Module Details

Modules are listed alphabetically by module name. Further information, including general descriptions, timetable and room information, and links to module organisers' web pages, is available on the departmental website. Timetable and room information is also available on the student notice boards.

MTH6100 Actuarial Mathematics
Organiser Dr R Harris
Level 6 **Credit value** 15 **Semester** B
Assessment 10% coursework, 90% final exam
Prerequisites MTH4101/MAS125 Calculus II, MTH5118/MAS228 Probability II
Overlaps MAS224 Actuarial Mathematics

Syllabus

1. Compound interest: discounting, force of interest, nominal values (APR); annuities certain: accumulated amount; schedule of principal and interest; perpetuities.
2. Life tables (LT): LT functions; the LT as model of cohort experience or stationary distribution; survival probabilities in terms of LTs. Reference to actual populations: tables of annuitants and assured lives. Select LTs.
3. Valuation: monetary functions; values of endowments, annuities and assurances.
4. Calculation of premiums; policy and surrender values; paid up policies.

Books
Reading List

- McCutcheon & Scott, An Introduction to the Mathematics of Finance (Heinemann)
- A Neill, Life Contingencies (Heinemann)
- Bowers, Gerber, Hickman et al., Actuarial Mathematics (SoA)
- Pollard, Mathematical Models for the Growth of Human Populations (CUP)

MTH703U Advanced Cosmology
Organiser Professor J E Lidsey
Level 7 **Credit value** 15 **Semester** A
Assessment 100% final exam
Prerequisites MTH5106/MAS226 Dynamics of Physical Systems
Overlaps MAS401 Advanced Cosmology

Syllabus

- Observational basis for cosmological theories. Derivation of Friedmann models and their properties.
- Cosmological tests: source counts; flux-redshift and luminosity-volume diagrams; integrated background radiation; evolution.
- Observational cosmology: the distance scale and the Hubble constant; the age of the Universe; the density parameter.
- Physics of the Big Bang: cosmological nucleo-synthesis; the cosmic microwave background radiation (CMBR); the decoupling era; large and small scale anisotropy in the CMBR; galaxy formation: the growth of fluctuations; effect of hot and cold dark matter; viable galaxy formation scenarios.
- The very early Universe: phase transitions; inflation; cosmic strings.
- The intergalactic medium: evidence from X-ray and microwave backgrounds, absorption-line systems in quasar spectra; Gunn-Peterson effect; role of dust in pre-galactic era; the Sunyaev-Zel'dovich effect.

MTH6103 Advanced Statistics Project

Organiser Dr L Pettit
Level 6 **Credit value** 30 **Semester** A and B
Assessment Written report, presentation and (possibly) oral exam
Prerequisites MTH5120/MAS232 Statistical Modelling I. At least another 30 credits of level-6 statistics. Before registering you must consult the module organiser.
Overlaps MAS332 Advanced Statistics Project

Syllabus The major part of this module is an individual project on some aspect of probability or statistical theory or applied statistics. It must be your own work in the sense that it gives an original account of the material, but it need not contain new mathematical results. The length should be the equivalent of between 3,500 and 7,000 words.
There will also be classes, which will cover the following:

1. Introduction to project work; development of a project proposal.
2. Statistical study skills, including use of literature, selection of appropriate methods of data analysis, selection of appropriate computer software.
3. Report writing.

The project will be assessed primarily by a written report and, at the examiners' discretion, an oral examination, but also by a presentation of 20–30 minutes in duration, to be given towards the end of semester 6. The contribution of the presentation will be on a sliding scale that will never decrease the project mark by more than 10% or increase it by more than 20%, and provided you make a reasonable attempt at giving a presentation it will not decrease your project mark.

MTH5100 Algebraic Structures I

Organiser Professor R A Wilson
Level 5 **Credit value** 15 **Semester** B
Assessment 10% coursework, 10% in-term test, 80% final exam
Prerequisites MTH4104/MAS117 Introduction to Algebra
Overlaps MAS201 Algebraic Structures I

Syllabus

1. Revision of sets, functions, operations, relations, equivalence relations.
2. Definition of group. Examples: permutation groups, matrix groups, groups of symmetries of regular polygons. Cyclic groups and their structure. Subgroups, subgroup test. Cosets, Lagrange's theorem, index. Multiplication table, Cayley's Theorem.
3. Homomorphisms, isomorphisms, automorphisms, with examples. Image and kernel. Normal subgroups. Construction of factor groups, correspondence theorem, isomorphism theorems. Direct products.
4. Definition of ring. Examples: matrix rings, residue class rings, division rings, fields. Gaussian integers. Integral domains, zero divisors, units, groups of units, examples. Euclidean functions, Euclidean domains, unique factorisation domains. Subrings, subring test.
5. Homomorphisms. Image and kernel. Ideals. Construction of factor rings. Correspondence and isomorphism theorems. Generators for ideals. Principal ideal domains, maximal ideals. Polynomial rings. Construction of fields as factor rings. Finite fields.

Books
Reading list

- P J Cameron, Introduction to Algebra (Oxford)

MTH6104 Algebraic Structures II

Organiser Dr J N Bray
Level 6 **Credit value** 15 **Semester** A
Assessment 10% coursework, 90% final exam
Prerequisites MTH5100/MAS201 Algebraic Structures I
Overlaps MAS305 Algebraic Structures II

Syllabus

1. Review of elements of groups and rings.
2. Group theory: group actions; finite p-groups; Sylow theorems and applications; Jordan-Holder theorem; finite soluble groups.
3. Ring theory: matrix rings; Noetherian rings and Hilbert's basis theorem.
4. Modules: foundations of module theory; isomorphism theorems; structure of finitely generated modules over Euclidean domains.

Books
Main text

- P J Cameron, Introduction to Algebra (OUP)

Other text

- W Ledermann and A J Weir, Introduction to Group Theory, second edition (Longman)

MTH6105 Algorithmic Graph Theory

Organiser Dr P Keevash
Level 6 **Credit value** 15 **Semester** B
Assessment 10% coursework, 90% final exam
Prerequisites MTH4108/MAS108 Probability I or MTH4104/MAS117 Introduction to Algebra
Overlaps MAS236 Algorithmic Graph Theory, MAS210 Graph Theory and Applications

Syllabus

1. Basics definitions and results: walks, paths cycles, connectedness, trees.
2. Applications of trees: finding connected components, depth and breadth first search, minimum weight spanning trees, shortest path spanning trees, longest path spanning trees in acyclic directed networks.
3. Maximum flows in networks.
4. Maximum size and maximum weight matchings in bipartite graphs.
5. Euler tours in graphs and digraphs and the Chinese Postman Problem.

Books
Main text

- A printed detailed course summary will be available from the Bookshop and/or the web.

Other texts

- Gibbons, Algorithmic Graph Theory, Cambridge University Press
- Wilson and Watkins, Graphs, An Introductory Approach, Wiley

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MTH705U Applied Statistics

Organiser Dr B Bogacka
Level 7 **Credit value** 15 **Semester** A
Assessment 3 reports (about 10–15 pages each, on separate topics), 33% each
Prerequisites MTH5120/MAS232 Statistical Modelling I required. MTH6116/MAS314 Design of Experiments, MTH6134/MAS339 Statistical Modelling II and MTH6139/MAS328 Time Series all useful.
Overlaps MAS421 Applied Statistics

Syllabus The semester will be divided into three four-week 'months'. In each month there is a genuine piece of applied statistics, led by a different lecturer. The lecturer will set it up with at most 2 lectures. At the end of the month the student will hand in a report of 10–15 pages. Statistical techniques and statistical computing packages from previous statistics modules will be needed. The three topics will be chosen from the following list:

1. Design of experiments
2. Medical statistics
3. Time series analysis of spacecraft data
4. Multivariate data from crop research

5. Agricultural statistics
6. Economic statistics
7. Industrial statistics

See module organiser before registering.

MTH707U Astrophysical Fluid Dynamics

Organiser Dr S V Vorontsov
Level 7 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH5102/MAS204 Calculus III, MTH6129/MAS229 Oscillations, Waves and Patterns, or equivalent
Overlaps MAS402 Astrophysical Fluid Dynamics

Syllabus

1. Basic fluid equations: Newtonian gravity; energy equations; thermodynamics; virial theorem.
2. Simple models of astrophysical fluids and their motions; linear perturbations about equilibrium; waves; typical scales.
3. Jeans instability and star formation.
4. Theory of rotating bodies.
5. Spherically symmetric accretion; Bernoulli's theorem; Bondi problem.
6. Viscous accretion discs; thin discs; steady discs.
7. Radial oscillation of stars; linear adiabatic wave equation; bounds on frequencies; non-adiabatic oscillations; quasi-adiabatic approximation.
8. Linear adiabatic non-radial oscillations; mode classification; JWKB method.
9. Helioseismology; asteroseismology.
10. Nonlinear acoustic waves; shocks; self-similarity solution for supernova blast wave.

MTH708U Astrophysical Plasmas

Organiser Professor D H Burgess
Level 7 **Credit value** 15 **Semester** A
Assessment 100% final exam
Prerequisites Consult the module organiser
Overlaps MAS429 Astrophysical Plasmas

Syllabus A plasma is an ionized gas where the magnetic and electric field play a key role in binding the material together. Plasmas are present in almost every astrophysical environment, from the surface of pulsars to the Earth's ionosphere. This course explores the unique properties of plasmas, such as particle gyration and magnetic reconnection. The emphasis is on the plasmas found in the Solar System, from the solar corona and solar wind to the outer reaches of the heliosphere and the interstellar medium. Fundamental astrophysical processes are explored, such as the formation of supersonic winds, magnetic energy release, and plasma coupling. The course highlights the links between the plasmas we can observe with spacecraft and the plasmas in more distant and extreme astrophysical objects.

MTH709U Bayesian Statistics

Organiser Dr L Pettit
Level 7 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH6136 Statistical Theory/MAS230 Fundamentals of Statistics II or MTH736U Mathematical Statistics; MTH5120/MAS232 Statistical Modelling I will be helpful
Overlaps MAS442 Bayesian Statistics

Syllabus

1. The Bayesian paradigm – likelihood principle, sufficiency and the exponential family, conjugate priors, examples of prior to posterior analysis, mixtures of conjugate priors, non-informative priors, two sample problems, predictive distributions, constraints on parameters, point and interval estimation,hypothesis tests, nuisance parameters.
2. Linear models – use of non-informative priors, normal priors, two and three stage hierarchical models, examples of one way model, exchangeability between regressions, growth curves, outliers and influential observations.
3. Approximate methods – normal approximations to posterior distributions, Laplace's method for calculating ratios of integrals, Gibbs sampling, finding full conditionals, constrained parameter and missing data problems, graphical models. Advantages and disadvantages of Bayesian methods.
4. Examples – appropriate examples will be discussed throughout the course. Possibilities include epidemiological data, randomised clinical trials, radiocarbon dating.

Books
Main texts

- Lee P M, Bayesian Statistics: An Introduction, (3rd Ed) Edward Arnold

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MTH4100 Calculus I
Organiser Dr R Klages
Level 4 **Credit value** 15 **Semester** A
Assessment 20% in-term test, 80% final exam
Prerequisites A-Level Mathematics or equivalent
Overlaps MAS115 Calculus I, ECN114 Math. Methods in Economics and Business I

Syllabus

1. Real numbers and the real line. Manipulation of algebraic equations involving the square root and modulus functions. Proving and solving algebraic inequalities. Triangle inequality. Trigonometric functions. Manipulation of trigonometric identities. Use of addition and double-angle formulas for expressing values of trigonometric functions in the surds form.
2. Functions and their graphs. Piecewise defined functions. Odd and even functions. Periodic functions. Monotone functions. Functions defined implicitly. Composition of functions and functional inverse. Scaling (vertical and horizontal), reflecting and shifting a graph; combinations of transformations on graphs.
3. Limits. Calculating limits using the Limit Laws and by eliminating zero denominators algebraically. The precise definition of a limit, finding deltas algebraically for given epsilons. One-sided limits and limits at infinity. Limits involving sin x/x. Limits at infinity of rational functions, horizontal and oblique asymptotes. Infinite limits and vertical asymptotes. Continuous functions. Intermediate Value Theorem for continuous functions (without proof) and its applications.
4. Derivatives I. Tangents and derivatives. Instantaneous rate of change (derivatives at a point). Derivative as a function. Calculating derivatives from the definition. One-sided derivatives. Basic rules of differentiation. Derivatives of trigonometric functions. The chain rule and parametric equations. Second-order derivatives. Finding derivatives using table of derivatives and rules of differentiation, including repeated use of the chain rule. (Technical dexterity of finding derivatives will be checked using test assessments.)
5. Derivatives II. Linearisation and differentials, estimating with differentials. Extreme values of functions. The Mean Value Theorem. Monotone functions and the First Derivative Test. Concavity and the Second Derivative Test. Curve sketching.
6. Derivatives III. Indeterminate forms $(0/0, \infty/\infty, \infty - \infty)$ of limits and l'Hopital's Rule. Limits involving $(1 + 1/x)^x$. Logarithmic, power and exponential rates of growth. Implicit differentiation. Tangent and normal to a curve defined by $f(x, y) = 0$ or parametrically. Derivatives of inverse functions. Inverse trigonometric functions (graphs, derivatives). First look at hyperbolic functions (algebra of, inverse functions, derivatives, graphs).
7. Integration I. Indefinite integral as anti-derivative. Techniques for evaluating indefinite integrals. Integration by substitution and integration by parts as the reverse processes of the

chain rule and product rules. [Non-examinable reading assignment for any student not taking MTH4102 Differential Equations in the second semester: first order separable differential equations; first order linear differential equations by the method of integrating factor.]

8. Integration II. Definite integral as limit of Riemann sums. Properties of definite integrals. Area under the graph of a non-negative function. The Fundamental Theorem of Calculus, evaluation of definite integrals. Area between curves. Area of a circle. First look at the improper integrals: area of unbounded planar regions.
9. First look at polar coordinates. Graphing in polar coordinates.

Books
Main text

- Calculus I and II follow Thomas' Calculus and make use of an interactive maths web site MyMathLab, which is tied to the book. Buying this book in advance is not advisable. The book and access code for MyMathLab will be available at a discounted price in September from the QM bookshop.

MTH4101 Calculus II

Organiser Professor C D Murray
Level 4 **Credit value** 15 **Semester** B
Assessment 20% in-term test, 80% final exam
Prerequisites MTH4100/MAS115 Calculus I
Overlaps MAS125 Calculus II, ECN114 Math. Methods in Economics and Business I, ECN124 Math. Methods in Economics and Business II

Syllabus

1. Complex numbers I: definition and their necessity for elementary operations, geometric representation, loci and regions in the complex plane, quadratic equations with real coefficients.
2. Complex numbers II: Euler's relation, DeMoivre's Theorem and applications to trigonometric identities, square root and log functions. Application to integrals $\int e^{ax}\cos(bx)dx$.
3. Derivatives IV: Functions of two variables. Domain, range and level curves. Graphing a function of two variables. Limits and continuity in the xy-plane. Partial derivatives. Statement and use of "mixed derivatives theorem" without proof. Differentiability and continuity and related theorems without proof. Chain rule for functions of two and three variables. Implicit differentiation revisited (Implicit Function Theorem).
4. Derivatives V. Directional derivatives and gradient vector. Tangent planes and normal lines. Estimating change in a specific direction. Linearization and differentials. Extreme values and saddle points. Lagrange multipliers.
5. Integration III. Double integrals as volumes under surfaces and areas in the plane. Properties of double integrals. Double integrals as repeated integrals, rectangular regions, simple non-rectangular regions.
6. Integration IV. Double integrals over non-rectangular regions, bounded and unbounded. Determining limits of integration. Reversing the order of integration. Transformations of variables – maps, domains, one-to-one maps, inverse maps.
7. Integration V. Change of variables in double integrals. Jacobians. Transformation to polar coordinates. Other transformations. Applications to normal distribution in probability. Triple and multiple integrals. Change of variables of integration. Applications of integrals.
8. Series I. Infinite sequences. Converging sequences. Diverging to infinity sequences. Calculating limits of sequences, including use of l'Hopital's rule. Infinite series (n-th term, partial sum, convergence, sum). Examples of converging series. Examples of diverging series. The n-th term test for divergence.
9. Series II. Series of positive terms and the Integral Test for convergence (second look at improper integrals). The ratio test.
10. Series III. Power series and convergence. Term-by-term differentiation and integration. Power series in the complex plane including existence of radius of convergence. Taylor and Maclaurin series and polynomials (via integration by parts). First look at the remainder term. Taylor series for common transcendental functions (exp, log, sin, cos, square root, cosh and sinh). Examples of applications of power series (power series solutions of differential equations, evaluating non-elementary integrals, evaluating indeterminate forms).

Books
Main text

- Calculus I and II follow Thomas' Calculus and make use of an interactive maths web site MyMathLab, which is tied to the book. Buying this book in advance is not advisable. The book and access code for MyMathLab will be available at a discounted price in September from the QM bookshop.

MTH5102 Calculus III

Organiser Dr W J Sutherland
Level 5 **Credit value** 15 **Semester** A
Assessment 10% coursework, 10% in-term test, 80% final exam
Prerequisites MTH4101/MAS125 Calculus II, MTH4103/MAS114 Geometry I
Overlaps MAS204 Calculus III

Syllabus

1. Arc-length of plane curves: length of a parametric curve, length of a curve $y = f(x)$. Length of the circumference of a circle, ellipse. Area and length in polar coordinates.
2. Vector fields, line, surface and volume integrals.
3. Grad, div and curl operators in Cartesian coordinates. Grad, div, and curl of products etc. Vector and scalar forms of divergence and Stokes's theorems. Conservative fields: equivalence to curl-free and existence of scalar potential. Green's theorem in the plane.
4. Orthogonal curvilinear coordinates; length of line element; grad, div and curl in curvilinear coordinates; spherical and cylindrical polar coordinates as examples.
5. A first look at Legendre polynomials.
6. Fourier series: full, half and arbitrary range series. Parseval's Theorem.
7. Laplace's equation. Uniqueness under suitable boundary conditions. Separation of variables. Two-dimensional solutions in Cartesian and polar coordinates. Axisymmetric spherical harmonic solutions.

Books
Main text

- Thomas' Calculus, 11th Edition (Addison Wesley)

Other texts

- M R Spiegel, Vector Analysis (Schaum Outline Series, McGraw-Hill)
- S Simons, Vector Analysis for Mathematicians, Scientists & Engineers (Pergamon Press)

MTH6107 Chaos and Fractals

Organiser Professor F Vivaldi
Level 6 **Credit value** 15 **Semester** A
Assessment 100% final exam
Prerequisites MTH4103/MAS114 Geometry I, MTH4101/MAS125 Calculus II
Overlaps MAS308 Chaos and Fractals

Syllabus

1. Continuous-time and discrete-time dynamical systems, Poincaré surface of section.
2. Fixed points, periodic orbits and their stability, 1-dimensional diffeomorphisms and their periodic orbits. Sharkovsky's theorem.
3. The logistic map, period-doubling scenario, Feigenbaum constants and Feigenbaum-Cvitanovic equation, tangent bifurcation and intermittency.
4. Definition of chaos, Lyapunov exponents, Bernoulli shift, topological conjugacy, symbolic dynamics.
5. Invariant measures and invariant densities, Perron-Frobenius operator, time and ensemble average, ergodicity.
6. Higher-dimensional maps, Jacobian matrix and stability of periodic orbits.
7. Examples of simple fractals, fractal dimension, Renyi dimensions.

8. Complex dynamics, Julia sets and Mandelbrot set, iterated function systems.

Books
Main text

- R Devaney, An introduction to chaotic dynamical systems (Addison-Wesley)

Other texts

- M Barnsley, Fractals Everywhere (Academic Press)
- Beck/Schloegl, Thermodynamics of Chaotic Systems (CUP)
- D Gulick, Encounters with Chaos (McGraw Hill)

MTH6108 Coding Theory

Organiser Dr M Fayers
Level 6 **Credit value** 15 **Semester** B
Assessment 10% coursework, 90% final exam
Prerequisites MTH5112/MAS212 Linear Algebra I
Overlaps MAS309 Coding Theory

Syllabus The concept of an error-correcting code is a very important one, with wide applications in communications. This module approaches the subject from a pure-mathematics perspective, to give the student a thorough grounding in construction of codes and decoding algorithms, and the main coding theory problem.

1. Basic concepts: codes, minimum distance, equivalence of codes.
2. The Main Coding Theory Problem. The Hamming, Singleton and Plotkin bounds.
3. Noisy channels and nearest-neighbour decoding. Statement of Shannon's Theorem.
4. Linear codes. Generator matrices and parity-check matrices. Syndrome decoding. (A very brief review of the required linear algebra will be given.)
5. Examples: Hamming, Reed-Muller and MDS codes. The Gilbert-Varshamov bound.

Books
Main text

- R Hill, A First Course in Coding Theory (OUP)

Other text

- J H van Lint, Introduction to coding theory (Springer)

MTH6109 Combinatorics

Organiser Professor T W Müller
Level 6 **Credit value** 15 **Semester** A
Assessment 10% coursework, 90% final exam
Prerequisites MTH5112/MAS212 Linear Algebra I required. MTH4108/MAS108 Probability I useful.
Overlaps MAS219 Combinatorics

Syllabus

1. Counting, binomial coefficients, recurrence relations, generating functions, partitions and permutations, finite fields, Gaussian coefficients.
2. Steiner triple systems, necessary conditions, direct and recursive constructions, structural properties and characterisations.
3. Ramsey's theorem, illustrations, proof and applications.
4. Transversal theory, Latin squares, Hall's theorem, upper and lower bounds.

Books
Main text

- P J Cameron, Combinatorics (CUP)

Other texts

- J H Van Lint, R M Wilson, A Course in Combinatorics (CUP)

- I Anderson, A First Course in Combinatorial Mathematics (OUP)
- N L Biggs, Discrete Mathematics, Oxford Science Publication (OUP)

MTH6110 Communicating and Teaching Mathematics: the Undergraduate Ambassadors Scheme

Organiser Dr C Agnor
Level 6 **Credit value** 15 **Semester** B
Assessment Journal of teaching activity (2,500 words) 30%; end-of-module report on "special project" (2,500 words) 30%; end-of-module presentation on "special project" (10 minutes plus 5 minutes discussion) 20%; teacher's end-of-module report 20%
Prerequisites Acceptance based on academic record and an interview in Semester A; consult the module organiser for details. Restricted to Queen Mary Mathematical Sciences students.

Syllabus Students will typically begin by observing the teacher's handling of the class and progress from this classroom assistant stage through small teaching tasks to at least one opportunity to undertake whole class teaching, possibly for a short part of a lesson. They will represent and promote mathematics as a potential university choice.
Students will undertake and evaluate a special project on the basis of discussion with the teacher. This may involve a specific in-class teaching problem or an extra-curricular project such as a lunchtime club or special coaching periods for higher ability pupils. The student will keep a journal of their own progress in working in the classroom environment, and they will be asked to submit a reflective written report on the special project and other relevant aspects of the school placement experience. This format is standard within the Undergraduate Ambassadors Scheme (http://www.uas.ac.uk/).

- Initial day of training.
- Competitive interview system to ensure students' suitability for the module.
- Student will be matched with an appropriate school and a specific teacher in the local area.
- Student will spend the equivalent of half a day a week in the school every week for a semester.
- No formal lectures.
- A supporting tutorial for one hour once a week for students to share experiences.
- Teachers will act as the main source of guidance but students will also be able to discuss their progress with the module organiser and the More Maths Grads team as needed.
- End of unit presentation of special project (15 minutes per student)

Books
You should consult:

- The National Curriculum website: http://www.nc.uk.net/home.html
- The Teacher Training Agency website: http://www.canteach.gov.uk
- The National Centre for Excellence in the Teaching of Mathematics: http://www.ncetm.org.uk
- Improving Learning in Mathematics: challenges and strategies, Malcolm Swan, which can be downloaded from http://www.maths4life.org/content.asp?CategoryID=1068

MTH6111 Complex Analysis

Organiser Professor S R Bullett
Level 6 **Credit value** 15 **Semester** A
Assessment 10% coursework, 90% final exam
Prerequisites MTH5103/MAS205 Complex Variables, MTH5105/MAS221 Differential and Integral Analysis
Overlaps MAS310 Complex Functions

Syllabus A rigorous module in complex analysis. The first part of the module will be concerned with detailed analysis of topics already seen in Complex Variables:

1. Differentiation and integration
2. Cauchy's theorem, Taylor and Laurent series

3. Conformal mappings and harmonic functions
4. The residue theorem and the calculus of residues

The second part of the module will introduce more advanced topics, e.g. some or all of

1. Riemann surfaces
2. Complex gamma, beta and zeta functions
3. Elliptic functions
4. Picard's theorem

Books
Consult module organiser before buying any book specifically for this module since we shall be using a number of texts. Possibilities include

- I N Stewart & D O Tall, Complex Analysis (CUP)
- H A Priestley, Introduction to Complex Analysis (OUP)

MTH5103 Complex Variables

Organiser Dr K Malik
Level 5 **Credit value** 15 **Semester** B
Assessment 10% coursework, 10% in-term test, 80% final exam
Prerequisites MTH4101/MAS125 Calculus II
Overlaps MAS205 Complex Variables

Syllabus

1. Complex numbers, functions, limits and continuity.
2. Complex differentiation, Cauchy-Riemann equations, harmonic functions.
3. Sequences and series, Taylor's and Laurent's series, singularities and residues.
4. Complex integration, Cauchy's theorem and consequences, Cauchy's integral formulae and related theorems.
5. The residue theorem and applications to evaluation of integrals and summation of series.
6. Conformal transformations.

Books
Other texts

- M R Spiegel, Complex Variables (Schaum Outline)
- R V Churchill & J.W. Brown, Complex Variables and Applications (McGraw Hill)
- H A Priestley, Introduction to Complex analysis (OUP)
- I N Stewart and D O Tall, Complex Analysis (Cambridge University Press)
- G Cain, `http://www.math.gatech.edu/~cain/winter99/complex.html` (Complex Analysis)
- Tristan Needham, Visual Complex Analysis (Oxford University Press)

MTH731U Computational Statistics

Organiser Dr H Grossmann
Level 7 **Credit value** 15 **Semester** B
Assessment 30% coursework, 70% final exam.
Prerequisites MTH6136 Statistical Theory/MAS230 Fundamentals of Statistics II or MTH736U Mathematical Statistics
Overlaps MAS344 Computational Statistics, MTH6114/MAS311 Computational Techniques in Statistics

Syllabus The techniques developed will be applied to a range of problems arising in business, economics, industry and science. Data analysis will be carried out using the user-friendly, but comprehensive, statistics package R.

1. Probability density functions: the empirical cdf; $q-q$ plots; histogram estimation; kernel density estimation.
2. Nonparametric tests: permutation tests; randomisation tests; link to standard methods; rank tests.

3. Data splitting: the jackknife; bias estimation; cross-validation; model selection.
4. Bootstrapping: the parametric bootstrap; the simple bootstrap; the smoothed bootstrap; the balanced bootstrap; bias estimation; bootstrap confidence intervals; the bivariate bootstrap; bootstrapping linear models.

Books
Main text

- J Gentle, Elements of Computational Statistics (Springer)

MTH5104 Convergence and Continuity

Organiser Dr M Walters
Level 5 **Credit value** 15 **Semester** A
Assessment 10% coursework, 10% in-term test, 80% final exam
Prerequisites MTH4100/MAS115 Calculus I
Overlaps MAS111 Convergence and Continuity

Syllabus

1. Real numbers: Algebraic properties, inequalities, supremum and infimum, completeness axiom for the existence of the supremum.
2. Sequences: Definition of limit and its use in specific examples, limit of sum, product and quotient of sequences. Bounded monotone sequences. Bolzano-Weierstrass Theorem.
3. Series: Convergent series, geometric series, harmonic series. Alternating series, comparison and ratio tests. Absolutely convergent series. Power series, radius of convergence. Examples, including sin(x), cos(x) and exp(x).
4. Real functions: Definition of limit, properties of limits.
5. Continuous functions: Definition of continuity and its use in specific examples, sum of continuous functions, composites of continuous functions (proofs), products/quotients of continuous functions (stated). Briefly, the Intermediate Value Theorem, application to roots of polynomials, boundedness of continuous functions on closed bounded intervals.
6. Definition of derivative. Continuity of differentiable functions.

Books
Main texts

- R Haggarty, Fundamentals of Mathematical Analysis (2nd ed.), Prentice-Hall, 1993, chapter 2 (going easy on axioms), chapters 3–5, chapter 6 (part).
  NB: Also very useful for MTH5105 (Differential and Integral Analysis) covering all material except convergence of sequences of functions.
- J M Howie, Real Analysis, Springer, 2001, chapters 1–3, chapter 4 (part)

MTH6115 Cryptography

Organiser Dr J N Bray
Level 6 **Credit value** 15 **Semester** B
Assessment 20% coursework, 80% final exam
Prerequisites MTH4104/MAS117 Introduction to Algebra, MTH5112/MAS212 Linear Algebra I
Overlaps MAS335 Cryptography

Syllabus

1. History and basic concepts: substitution and other traditional ciphers; plaintext, ciphertext, key; statistical attack on ciphers.
2. One-time pad and stream ciphers: Shannon's Theorem; one-time pad; simulating a one-time pad; stream ciphers, shift registers.
3. Public-key cryptography: basic principles, including brief discussion of complexity issues; knapsack cipher; RSA cipher; digital signatures.

Examples of optional topics which may be included: secret sharing, quantum cryptography, the Enigma cipher

Books
Reading list

- Simon Singh, The Code Book: How to Make It, Break It, Hack It, or Crack It, Delacorte Press (introductory)
- Dominic Welsh, Codes and Cryptography, Oxford University Press
- Paul Douglas Stinson, Cryptography: Theory and Practice (Chapman and Hall)

MTH6116 Design of Experiments

Organiser Dr H Maruri-Aguilar
Level 6 **Credit value** 15 **Semester** B
Assessment 20% in-term tests, 80% final exam
Prerequisites MTH6134/MAS339 Statistical Modelling II
Overlaps MAS314 Design of Experiments

Syllabus Real life experiments will be discussed from several applications in business, especially market research, industry and science, including medicine.

1. Treatment structure: factors, main effects, interaction.
2. Completely randomised designs.
3. Blocking.
4. Row-column designs.
5. Experiments on people and animals.
6. Nested blocks, split-plot designs.
7. General orthogonal designs.
8. Incomplete-block designs.
9. Factorial designs in incomplete blocks.

Seven or eight lectures will be replaced by discussion sessions, when students present their solutions to assignments. Solutions are discussed by the whole class because most questions have no single correct answer.

Books
Reading list

- Cox, The Planning of Experiments (Wiley)
- John, Statistical Design & Analysis of Experiments (MacMillan)
- Kempthorne, The Design & Analysis of Experiments (Wiley)
- Cochran/Cox, Experimental Design (Wiley)
- Clarke/Kempson, Introduction to the Design & Analysis of Experiments (Arnold)

MTH4102 Differential Equations

Organiser Professor J E Lidsey
Level 4 **Credit value** 15 **Semester** B
Assessment 20% in-term tests, 80% final exam
Prerequisites MTH4100/MAS115 Calculus I, MTH4103/MAS114 Geometry I
Overlaps MAS118 Differential Equations, MAS112 Modelling Dynamical Systems

Syllabus

1. Revision of geometrical meaning of derivative, anti-derivative. Differentiation of combined and composed functions. Verification of solution of differential equation by substitution. Particular and general solutions. The role of initial or boundary conditions. Solution of simplest ODEs by direct integration. Separation of variables for first order differential equations, implicitly defined solutions.
2. First order linear differential equation (integrating factors), homogeneous and inhomogeneous equations.
3. Differential forms, integral curves, exact differential equations.

4. Interpretation of first order differential equation in terms of direction fields, the initial value problem, solution by geometric method.
5. Linear second order differential equations with constant coefficients, homogeneous equations, superposition, characteristic equations, real roots (including degenerate equal roots case), complex roots.
6. Inhomogeneous equations with constant coefficients, method of undetermined coefficients, variation of constants formula, forced oscillations and visualisation.
7. Matrices, eigenvalues and eigenvectors (2-dimensional).
8. Linear systems in two dimensions, reduction of linear second order ordinary differential equation to a linear system in two variables. Various types of solution in terms of exponential functions.
9. Phase space for two dimensional linear systems, stable/unstable nodes/foci, planar phase space portraits, classification of equilibria. Stability and instability of autonomous linear equations, characterisation of equilibrium points in terms of stability. Nonlinear systems – finding fixed points and their linearisations.
10. The Linearisation Theorem and examples. Linearisation breakdown by examples.

Books
Main text

- J Polking, A Boggess, D Arnold: Differential Equations (Pearson 2006), ISBN 0–13–143738–0

MTH5105 Differential and Integral Analysis
Organiser Dr T Prellberg
Level 5 **Credit value** 15 **Semester** B
Assessment 10% coursework, 10% in-term test, 80% final exam
Prerequisites MTH5104/MAS111 Convergence and Continuity
Overlaps MAS221 Differential and Integral Analysis

Syllabus

1. Differentiable functions: Definition of differentiability. Algebra of derivatives, chain rule. Derivative of inverse function. Rolle's Theorem, Mean Value Theorem and applications. Taylor's Theorem.
2. Integration: Darboux definition of Riemann integral, simple properties. Continuous functions are integrable (via uniform continuity). Fundamental Theorem of the calculus, integral form of Mean Value Theorem and of the remainder in Taylor's Theorem; applications to some well known series (log, arctan, binomial). Improper integrals.
3. Sequences of functions: pointwise and uniform convergence. Weierstrass M-test. Term-by-term integration of power series.

Books
Main text

- R Haggarty, Fundamentals of Mathematical Analysis (Addison-Wesley)

Other texts

- J Stewart, Single Variable Calculus, (Brooks/Cole Publishing Company, 4th edition, 1999)
- C Clark, Elementary Mathematical Analysis (Wadsworth, 1982)
- M D Hatton, Mathematical Analysis (Hodder & Stoughton, 1977)
- B M Mitchell, Calculus (without analytic geometry) (Heath, 1969)

MTH5106 Dynamics of Physical Systems
Organiser Dr J Cho
Level 5 **Credit value** 15 **Semester** A
Assessment 10% coursework, 10% in-term test, 80% final exam
Prerequisites MTH4101/MAS125 Calculus II, MTH4102/MAS118 Differential Equations
Overlaps MAS226 Dynamics of Physical Systems

Syllabus Some topics may already have been met in A-level Physics or Mechanics.

- Review of motion in space: displacement, velocity and acceleration using vectors; equation of motion; concept of constants of motion, energy and potentials; circular motion (plane polar coordinates).
- Mathematical modelling skills; from statement of problem to mathematical model; testing and evaluating a mathematical model.
- Newton's laws of motion. Examples of different types of motion due to forces and force fields, including resistive forces, and restoring forces: springs, ice hockey and parachutists.
- Newtonian model of gravity; sphere theorem; projectile motion and escape speed; variable mass: footballs, rockets and black holes.
- Central forces (e.g. gravity and Coulomb electrostatic forces); conditions for conservative force; potentials and conservation of angular momentum; orbit theory: polar equation of motion, types of orbit, Kepler's Laws: planets, asteroids and impact hazards.

Books
Texts

- P Smith and R C Smith, Mechanics (Wiley)
- Phil Dyke & Roger Whitworth, Guide 2 Mechanics (Palgrave Mathematical Guides)

MTH6117 Entrepreneurship and Innovation

Organiser Enterprise Training, Research and Enterprise, City University – email: enquiries AT enterprisetraining.org.uk; QM contact Dr V Easson
Level 6 **Credit value** 15 **Semester** A
Assessment 25% report (group work), 25% essay (individual), 50% final exam
Prerequisites None, but restricted to Queen Mary Mathematical Sciences students.
Overlaps DCS341 Entrepreneurship in IT, PHY333 Entrepreneurship and Innovation

Syllabus The aim of this module is to increase your awareness of the commercial opportunities available to you in the area of mathematical sciences. We examine how to cultivate an entrepreneurial mind set and discuss the routes available for turning your ideas into business ventures. The module provides an introduction to a number of crucial business skills such as financial planning, business planning and how to sell yourself and your ideas.

- Highlight the importance of commercialisation of innovative ideas both in the university and the industrial environment.
- Creatively explore commercial opportunities within mathematics and science.
- Introduce the different routes available to take an idea to market.
- Develop the skills required to start a business venture.
- Explain the key considerations involved in intellectual property and idea protection.
- Introduce the key aspects of financial management required in the development of a business venture.

Assessment will include a group 5-page business plan and 10-minute presentation, and an individual 1,000 to 2,000 word essay/report.

Books
Optional texts

- Harvard Business Review on Entrepreneurship, Harvard Business School Press, 1999
- The Definitive Business Plan (2nd Edition), Richard Stutely, Prentice Hall, 2002
- Entrepreneurship and Small Business (2nd Edition), Paul Burns, Palgrave London, 2007

MTH3100 Essential Mathematical Skills

Organiser Semester 1: Dr K Malik; Semester 2: Dr O Bandtlow
Level 3 **Credit value** 0 **Semester** A and B
Assessment 100% multiple choice exam
Prerequisites None
Overlaps MAS010 Essential Mathematical Skills

Syllabus

1. Decompose an integer as a product of prime numbers
2. Calculate the GCD and LCM of a pair of integers
3. Compute quotient and remainder of integer division
4. Simplify arithmetical expressions involving fractions
5. Perform simple estimations
6. Compute quotient and remainder of polynomial division
7. Simplify polynomial and rational expressions
8. Simplify expressions involving square roots
9. Perform algebraic substitutions
10. Solve linear and quadratic equations and inequalities

Books
Main text

- Essential Mathematics `http://www.maths.qmul.ac.uk/~fv/books/em/embook.pdf` (web-book)

MTH735U Extrasolar Planets and Astrophysical Discs
Organiser Professor R P Nelson
Level 7 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH5102/MAS Calculus III, MTH5106/MAS226 Dynamics of Physical Systems

Syllabus
Extrasolar planets

- Detection techniques: Doppler method, transit method, direct detection, microlensing
- Statistical description of data: mass distribution; orbital properties; correlation with stellar metallicity, physical properties
- Properties of individual exoplanets and exoplanet systems
- Comparison with Solar System planets

Planetary system formation model

- Formation of protoplanetary discs during star formation
- Protoplanetary disc properties
- Dust coagulation, runaway growth, oligarchic growth
- Terrestrial planet formation via giant impacts
- Giant planet formation: core accretion model versus gravitational instability model
- Planet migration

Origin of life

- Definition of life
- Conditions required for emergence of life – the habitable zone
- Basic ideas about emergence of self-replicating molecules (RNA, DNA)

Accretion discs

- Basic accretion disc theory: angular momentum transport mechanisms; diffusion equation for evolution;
- origin of disc turbulence through the MRI
- Close binary systems: classification; the Roche potential; Cataclysmic Variables; low and high mass X-ray binaries; outburst phenomena
- Accretion discs in active galactic nuclei – observations and models

Books
Main texts

- Planetary Sciences, I de Pater & J J Lissauer
- Accretion Power in Astrophysics, J Frank, A King, D Raine

MTH711U Extremal Combinatorics

Organiser Dr J R Johnson
Level 7 **Credit value** 15 **Semester** A
Assessment 100% final exam
Prerequisites MTH6109/MAS219 Combinatorics and MTH6105/MAS236 Algorithmic Graph Theory will be helpful
Overlaps MAS444 Extremal Combinatorics

Syllabus

1. Extremal Graph Theory: introduction (what is an extremal problem/result, some simple examples). Cycles (Dirac's theorem). Complete Graphs (Turan's theorem). Zarankiewicz problem (bipartite analogue of Turan). Erdos-Stone theorem.
2. The Discrete Cube: Sperner's theorem. Shadows and isoperimetric inequalities (LYM inequality, the Kruskal-Katona theorem, Harper's theorem, edge isoperimetric inequality).
3. Intersecting Families: Erdos-Ko-Rado theorem. Katona's t-intersecting theorem. Brief discussion of uniform t-intersecting problem (with statement but not proof of Ahlswede and Khachatrian's complete intersection theorem). Modular intersections (Frankl-Wilson theorem and some extensions and applications).
4. Other Topics: other topics of a similar flavour chosen according to class interest and time.

Books

The lecture notes will be self contained. Examples of books giving background material and further reading are:

- B Bollobás, Combinatorics, Cambridge University Press, 1986
- B Bollobás, Modern Graph Theory, Springer-Verlag, 1998
- S Jukna, Extremal Combinatorics: With Applications in Computer Science, Springer-Verlag, 2001

MTH733U Fields and Galois Theory

Organiser Professor L H Soicher
Level 7 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH5100/MAS201 Algebraic Structures I
Overlaps MAS316 Galois Theory

Syllabus Where useful, the characteristic will be restricted to zero to simplify the development.

1. Field theory: prime fields and characteristic, theory of finite fields, simple field extensions, principal element theorem, degree of an extension, product rule for degree, splitting fields, automorphisms of field extensions, embedding of one field extension into another, separability, normal extensions, fundamental theorem of Galois theory.
2. Applications: insolubility of equations of degree ≥ 5 by radicals, equivalence with insolubility of the Galois group, specific examples of insoluble equations over the rationals, ruler and compass constructions, symmetric polynomials (are generated by elementary symmetric polynomials).

Books

Main text

- I N Stewart, Galois Theory (Chapman & Hall)

MTH737U Fluid Dynamics

Organiser Dr J Cho
Level 7 **Credit value** 15 **Semester** B
Assessment 10% coursework, 90% final exam
Prerequisites MTH4102/MAS118 Differential Equations, MTH5102/MAS204 Calculus III, MTH5106/MAS226 Dynamics of Physical Systems. This module is suitable for third-year undergraduates.
Overlaps MTH6119/MAS349 Fluid Dynamics

Syllabus

1. Introduction
    - Mathematical preliminaries: vector identities, integral theorems, tensors and index notation
    - Lagrangian and Eulerian descriptions, material derivative and stream line
    - Euler and vorticity equations, conservation of mass and momentum, equation of state
2. Viscosity
    - Reynolds number – swimming tadpoles, disappearing windows and galaxies
    - Poiseuille and boundary layer flows
    - Diffusion of shear and vorticity
3. Waves
    - Wave dispersion, dispersion relation, phase and group velocities, linearisation
    - Shallow- and deep-water waves, sound: tsunamis and shouting upwind
    - Planetary and gravity waves, the weather and the ozone hole
    - Nonlinear behaviour: characteristics, hydraulic jumps, shocks and solitons
4. Vortices and vorticity
    - Kelvin and Helmholtz theorems, vortex lines, pairs and shedding, flying
    - Vortex sheets and shear instability, billow clouds
5. Advanced topics (as time permits)
    - Instability: Kelvin-Helmholtz, Rayleigh, thermal, centrifugal, baroclinic, etc.

Books
Main text

- Acheson, Elementary Fluid Dynamics (OUP)

MTH6120 Further Topics in Mathematical Finance

Organiser Professor C Beck
Level 6 **Credit value** 15 **Semester** B
Assessment 10% coursework, 90% exam
Prerequisites MTH5118/MAS228 Probability II, MTH6121/MAS343 Introduction to Mathematical Finance
Overlaps MAS345 Further Topics in Mathematical Finance

Syllabus

1. Revision of: geometric Brownian motion; interest rates and present value analysis; the arbitrage theorem; the Black-Scholes Formula; properties of the Black-Scholes option cost; arbitrage strategy.
2. Additional results on option.
3. Valuing by expected utility.
4. Deterministic and probabilistic optimisation models.
5. Exotic options.
6. Some examples beyond geometric Brownian motion models.
7. Autoregressive models and mean reversion.

Books
Main text

- Sheldon M Ross, An elementary introduction to Mathematical Finance: Options and other topics, Cambridge University Press (ISBN 0–521–81429–4)

MTH4103 Geometry I

Organiser Professor L H Soicher
Level 4 **Credit value** 15 **Semester** A
Assessment 20% in-term tests, 80% final exam.
Prerequisites A-Level Mathematics or equivalent
Overlaps MAS114 Geometry I

Syllabus

1. Phrasebook up to $\mathbb{R}^3$.
2. Vectors in 2-space and 3-space, expressed as $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ or as row or column vectors. Addition of vectors. Length of vectors.
3. Vector and cartesian equations of a straight line in $\mathbb{R}^2$ and $\mathbb{R}^3$.
4. Scalar multiple and scalar product of vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$. Cartesian equation of a plane in $\mathbb{R}^3$. Intersections of two or three planes. Solution of families of linear equations in x, y, z by reduction to echelon form.
5. Vector products in $\mathbb{R}^3$. Volume of parallelepiped as given by triple scalar product and determinant.
6. Linear transformations in $\mathbb{R}^2$, expressed by matrices with respect to the standard basis $\mathbf{i}, \mathbf{j}$. Examples: rotations, reflections, dilations, shears; their matrices.
7. In $\mathbb{R}^2$, characteristic equation, eigenvalues and eigenvectors, trace. Application to the examples in (6) (e.g. rotations with integer trace and the crystallographic restriction).
8. Extension of (6), (7) to $\mathbb{R}^3$.
9. Addition and multiplication of 2×2 and 3×3 matrices. Their interpretation as addition and composition of linear transformations. Inversion of matrices in $\mathbb{R}^2$ and in $\mathbb{R}^3$. (Examples and exercises may include 2×3 and 3×2 matrices.)

Books
Main text

- A E Hirst, Vectors in 2 or 3 dimensions, Elsevier 1995

Other texts

- In addition, Professor Chiswell's notes on Matrices and Geometry will be helpful for some parts of the course, and will be available online.

MTH5109 Geometry II: Knots and Surfaces

Organiser Dr J Elmer
Level 5 **Credit value** 15 **Semester** B
Assessment 10% coursework, 10% in-term test, 80% final exam
Prerequisites MTH4103/MAS114 Geometry I, MTH4101/MAS125 Calculus II
Overlaps MAS231 Geometry II: Knots and Surfaces

Syllabus

1. Knots and the unsolved problem of their classification. Reidemeister moves, Jones polynomial. Examples including trefoil, figure-eight.
2. Parametrised regular curves, their curvature and torsion defined by vector cross and dot products. Unit speed parametrisation and arc length.
3. Principal normal, co-normal and theorem that torsion and curvature can be prescribed up to rigid motions of $\mathbb{R}^3$.
4. Planar curves, signed curvature and the winding number theorem.
5. Surfaces, doughnuts and pretzels (classification by number of holes). Surface patches of smooth surfaces.
6. Orientability of a surface and unit normal. Examples of orientable and non orientable surfaces such as Möbius band.
7. Studying curves lying in surfaces. First fundamental form and area, second fundamental form, geodesic and normal curvatures.
8. Principal, mean and Gauss curvature of a surface. Elliptic, hyperbolic and parabolic points. Principal vectors and Euler's theorem.
9. Geodesics. Great circles on spheres and other examples.
10. Gauss-Bonnet theorem for integral of geodesic curvature in terms of integral of Gauss curvature in the interior, for simple closed curves and for curvilinear n-gons.
11. Discussion on hyperbolic surfaces and/or higher dimensional spaces.

Books
Main text

- A Pressley, Elementary Differential Geometry, Springer UMS 2000

MTH714U Group Theory

Organiser Professor R A Wilson
Level 7 **Credit value** 15 **Semester** A
Assessment 100% final exam
Prerequisites MTH6104/MAS305 Algebraic Structures II, or MTH5100/MAS201 Algebraic Structures I supplemented with some preliminary reading on groups
Overlaps MAS428 Group Theory

Syllabus

1. Revision of basic group theory, isomorphism theorems, Jordan-Holder theorem, Sylow's theorems, the structure theorem for finite abelian groups.
2. Permutation groups: transitivity, primitivity, symmetric and alternating groups. Maximal subgroups, wreath products, Iwasawa's Lemma. The outer automorphism of S_6.
3. Linear groups: finite fields, general linear groups, projective special linear groups. Projective lines and isomorphisms of some projective special linear groups with alternating groups. Simplicity of $PSL_n(q)$.

See module organiser before registering.

Books

The recommended text is J. L. Alperin and R. W. Bell, Groups and Representations, Springer (1995). Chapters 1 to 4 cover most (but not all) of what is in this module, plus a little extra, including the required background from Algebraic Structures II.

MTH4104 Introduction to Algebra

Organiser Dr I Tomasic
Level 4 **Credit value** 15 **Semester** B
Assessment 20% in-term tests, 80% final exam
Prerequisites MTH4103/MAS114 Geometry I required. MTH4108/MAS108 Probability I useful.
Overlaps MAS117 Introduction to Algebra

Syllabus

1. Mathematical basics: proofs, necessary and sufficient conditions, proofs and counterexamples, definitions, existence and uniqueness.
2. Numbers: integers, rationals, real numbers, complex numbers. Induction. Irrationality of $\sqrt{2}$. Polynomials, matrices.
3. Sets, subsets, functions, relations. One-to-one and onto functions. Equivalence relations and partitions.
4. Division algorithm and Euclidean algorithm. Modular arithmetic. Solving polynomials; remainder and factor theorems.
5. Rings and fields, ideals, factor rings.
6. Groups, subgroups, cyclic groups, Lagrange's Theorem.
7. Permutations, symmetric group, sign.

Books

Reading list

- D A R Wallace: Groups, Rings and Fields, Springer, London 1998; ISBN 3540761772
- A Chetwynd and P. Diggle: Discrete Mathematics, Butterworth-Heinemann, 1995

MTH715U Introduction to Dynamical Systems

Organiser Dr W Just
Level 7 **Credit value** 15 **Semester** A
Assessment 100% final exam
Prerequisites MTH6107/MAS308 Chaos and Fractals
Overlaps MAS424 Introduction to Dynamical Systems

Syllabus

- Dynamical systems in one and two dimensions.
- Computation of periodic orbits and their multipliers.
- Symbolic dynamics and one dimensional Markov maps.
- Lyapunov exponents, invariant distributions, and correlation functions.

MTH4105 Introduction to Mathematical Computing

Organiser Dr H Touchette
Level 4 **Credit value** 15 **Semester** A
Assessment 20% in-term tests, 80% final exam
Prerequisites A-Level Mathematics or equivalent
Overlaps MAS116 Introduction to Mathematical Computing

Syllabus

Part I – Interactive Mathematical Computing

1. Introduction to Maple: The Maple worksheet; online help; execution groups and text regions; basic computational number systems (integer, rational, float); simple arithmetic operations; factorial (!) and big numbers; Pi and numerical approximation using evalf; %; comma operator and expression sequences; command completion.
2. Continuous Mathematics: Variables, assignment and automatic evaluation; indeterminates and (univariate) polynomials; simple polynomial algebra; expand, factor, simplify; sqrt, exp, log and trigonometric functions; substitution and evaluation using eval; equations and inequalities; solve and fsolve; diff; int and evalf(Int ...); limit; series and taylor.
3. Discrete Mathematics: Integer arithmetic, divisibility and prime numbers: irem, iquo, igcd, ifactor, isprime; structured data: sequences, lists and sets; seq; nops; indexing using op and [ ]; index ranges; set operations; map; add, mul, sum.
4. Vectors, Matrices and Multivariate Algebra: Inputting row/column vectors and matrices; Vector and Matrix; vector and matrix algebra; scalar and vector product; exact and approximate eigenvalues and eigenvectors. Multivariate expressions; solving coupled multivariate equations.
5. Plotting and Tabulating: plotting univariate expressions; multiple plots; using the graphical user interface to read off intersections; lists of points; bivariate expressions as surfaces; 2D curves and 3D surfaces defined implicitly and parametrically; vectors; linear transformations; ellipses, ellipsoids and eigenvectors. Introduction to spreadsheets.

Part II – Mathematical Programming

1. Boolean Logic: Boolean constants (true, false); relational operators, evalb, is; use of evalf; Boolean operators (and, or, not); truth tables (using spreadsheets); Boolean algebra; analogy with set theory.
2. User-defined Functions: Arrow syntax; anonymous and named functions; polynomial and elementary transcendental examples; use with map; predicates (Boolean-valued functions); select and remove.
3. Repeated Execution: do ... end do; for ... to; while; for ... in; applications such as recursive sequences and iterative approximation, e.g. Iterative method for solving univariate equations, power method for largest eigenvalue; single/double loops over vector/matrix elements.
4. Conditional Execution: if ... then ... end if; else; elif; applications within loops (e.g. finding the maximum value in a list, vector or matrix and convergence of iterations); piecewise-defined functions; characteristic functions on sets; use with add.
5. Procedures: proc ... end proc; variable scope; local; global; return value versus side effects; return; error; print; applications such as base conversion, simple statistics.
6. Procedural Programming: The use of procedures for structuring programs; converting algorithms into programs; program design; debugging.

Books
You may find the following books useful

- F Vivaldi, Experimental Mathematics with Maple, Chapman & Hall, CRC Press 2001
- F J Wright, Computing with Maple, Chapman & Hall, CRC Press 2001

MTH6121 Introduction to Mathematical Finance

Organiser Dr D S Stark
Level 6 **Credit value** 15 **Semester** A
Assessment 10% coursework, 90% exam
Prerequisites MTH4108/MAS108 Probability I, MTH4101/MAS125 Calculus II
Overlaps MAS343 Introduction to Mathematical Finance

Syllabus

1. Pointers/revision of probability concepts: probability and events, conditional probability, random variables and expected values, covariance and correlation. Normal random variables and their properties, central limit theorem.
2. Pricing models; geometric Brownian motion and its use in pricing models. Brownian motion.
3. Interest rates and Present Value Analysis – including rate of return and continuously varying interest rates.
4. Pricing contracts via arbitrage – options pricing and examples.
5. The arbitrage theorem – proof and interpretation.
6. The Black-Scholes Formula. Properties of the Black-Scholes option cost. Arbitrage strategy.
7. A derivation of the Black-Scholes formula.

Books
Main text

- Sheldon M Ross, An elementary introduction to Mathematical Finance: Options and other topics, Cambridge University Press (ISBN 0–521–81429–4)

MTH5110 Introduction to Numerical Computing

Organiser Dr W Just
Level 5 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH4103/MAS114 Geometry I, MTH4105/MAS116 Introduction to Mathematical Computing, MTH4101/MAS125 Calculus II
Overlaps MAS235 Introduction to Numerical Computing

Syllabus This course investigates the use of computer algebra, numerical techniques and computer graphics as tools for developing the understanding and the solution of a number of problems in the mathematical sciences. The computer algebra system used for this course will be Maple.

1. Brief revision of Maple. Tracing algorithms and debugging.
2. Numerical and symbolic operations on matrices: obtaining and examining the properties of eigenvalues and eigenvectors.
3. Numerical and symbolic solution of algebraic equations.
4. Integration: overview of numerical techniques, symbolic generation of quadrature rules, comparison of numerical integration using numerical techniques and using symbolic analysis.

Books
Reading list

- F J Wright, Computing with Maple, Chapman & Hall/CRC (2001)

MTH4106 Introduction to Statistics

Organiser Professor R A Bailey
Level 4 **Credit value** 15 **Semester** B
Assessment 20% in-term tests, 80% exam
Prerequisites MTH4100/MAS115 Calculus I, MTH4108/MAS108 Probability I, MTH4101/MAS125 Calculus II (normally taken concurrently)
Overlaps MAS113/MTH4109/MAS113X Fundamentals of Statistics I, ECN104 Introductory Statistics for Economics and Business

Syllabus

1. Data and statistics. Types of variables. Simple plots, mean and median. Five number summary, box plots. Sample variance, inter-quartile range, skewness. Effect of linear transformations on summary statistics. Scatterplots and marginal plots. Sample correlation r and proof that $-1 < r < 1$. Introduction to Minitab.
2. Revision of discrete random variables (rvs). Probability generating function and its use (factorial moments, sums of independent random variables).
3. Ideas of statistical modelling, populations and samples. Random samples, sampling distribution of proportion. Point estimate of population proportion, unbiasedness.
4. Hypothesis tests on proportions, basic ideas, type I and II errors. One- and two-sided alternative hypotheses.
5. Continuous rvs. Cumulative distribution function and probability density function (pdf). Moments. Exponential and uniform distributions. Monotone transformations of rvs. Proof of pdf of new continuous rvs.
6. Normal rv, standard and general. Use of normal tables. Simulation – how to sample from different distributions, simulation of simple functions of rvs.
7. Introduction to joint distribution of two or more continuous rvs. Covariance, correlation and independence. Point estimation. Unbiasedness of sample mean and variance, calculation of bias.
8. Statements of law of large numbers and central limit theorem. Simulations of theoretical results. Linear combinations of normal rvs. Normal approximation to binomial and Poisson rvs, continuity correction. Sampling distributions of sample total, mean and variance.
9. 1-sample z test. Significance levels and p-values. Large sample applications.
10. Confidence intervals (CIs) – general ideas, CI for normal mean and large sample applications. Confidence intervals for a Poisson mean.
11. Conditioning on a continuous rv. Conditional expectation. Computing expectations by conditioning. Conditional variance.

Books
Main texts A book which suits YOU best to learn statistics is best (for you). You are encouraged to use it, whether it is one from the list below or another one.

- Devore, J L (2004). Probability and Statistics for Engineering and Sciences. Thomson Brooks/Cole. 6th Edition, Duxbury Press
- Wild, C J and Seber, G A F (2000). Chance Encounters. A First Course in Data Analysis and Inference. Wiley, New York
- Hines, W W and Montgomery, D C (1990). Probability and Statistics in Engineering and Management Science. Third Edition. Wiley
- Newbold, P (1988). Statistics for Business and Economics. Prentice-Hall International. New Jersey

You should already have a copy of

- Lindley, D V and Scott, W F (1995). New Cambridge Elementary Statistical Tables. Cambridge University Press for MTH4108/MAS108

MTH5112 Linear Algebra I
Organiser Dr O Bandtlow
Level 5 **Credit value** 15 **Semester** A
Assessment 10% coursework, 10% in-term test, 80% final exam
Prerequisites MTH4103/MAS114 Geometry I
Overlaps MAS212 Linear Algebra I

Syllabus

1. Systems of linear equations: Elementary row operations, solution by Gaussian elimination, echelon forms; existence/uniqueness of solutions.
2. Matrix algebra: Revision of matrix addition and multiplication from Geometry I. $A\mathbf{x}$ as a linear combination of the columns of A. Matrix inverse. Matrix transpose. Special types of square matrices. Linear systems in matrix notation. Elementary matrices and row operations. Reduced row echelon form for square matrices, conditions for non-singularity, matrix inverses by Gaussian elimination.

3. Determinants: Cofactors and row/column expansions. Elementary row/column operations. Determinant of matrix transpose, of product of matrices. Matrix inverse in terms of adjoint. Cramer's rule.
4. Vector spaces (over $\mathbb{R}$ and $\mathbb{C}$): Definition and examples. Subspaces. Spanning sets. Linear independence. Basis and dimension of a vector space. Change of basis. Row and column spaces, rank. The null space.
5. Linear Transformations: Definition and examples. Matrix representations of linear transformations. The law of change of matrix representation under a change of basis. The Rank-Nullity Theorem.
6. Orthogonality in $\mathbb{R}^n$: Scalar product – definition and properties. Orthogonal and orthonormal sets. Orthogonal complements. Orthogonal projections. Fundamental Subspace Theorem. Orthogonal matrices. Least-squares solutions of inconsistent systems. Gram-Schmidt process.
7. Eigenvalues and Eigenvectors: The equation $A\mathbf{x} = \lambda\mathbf{x}$. The characteristic polynomial. Eigenvalues and eigenvectors of special classes of matrices. Real symmetric matrices: orthogonal diagonalisation. Similarity: distinct eigenvalues and diagonalisation.

Books
Main text

- S J Leon: Linear Algebra with Applications. 7th Ed. (Pearson)

MTH6140 Linear Algebra II

Organiser Professor O Jenkinson
Level 6 **Credit value** 15 **Semester** A
Assessment 10% coursework, 90% final exam
Prerequisites MTH4104/MAS117 Introduction to Algebra, MTH5112/MAS212 Linear Algebra I
Overlaps MAS317 Linear Algebra II

Syllabus

1. Bilinear forms over finite dimensional real and complex vector space. Sesquilinear forms over complex vector spaces. Proof of Sylvester's law of inertia. Positive definite forms over real vector spaces, Hermitian forms over complex vector spaces.
2. Orthogonality, the Gram-Schmidt orthogonalisation process, orthogonal projections.
3. Revision of vector spaces, subspaces, eigenspaces, linear maps, direct sum, kernel and image, spanning set, linear independence, basis, dimension, Steinitz Exchange Lemma, dimension formula for subspaces, with rigorous proofs.
4. Properties of determinants and their connection with adjoints. The Cayley-Hamilton theorem and its proof. Eigenvalues, trace and determinant. Eigenvalues of a symmetric matrix.
5. Linear functional, dual spaces, equality of row and column rank of a matrix.
6. Symmetric, skew-symmetric and alternating bilinear forms over arbitrary fields. Skew-Hermitian forms over complex vector spaces.
7. Simultaneous diagonalisation, for linear map and positive definite symmetric form, and for two symmetric forms.

Books
Main text

- S Lipshutz, Linear Algebra (2nd edition) (Schaum Outline Series)

MTH6122 Linear Operators and Differential Equations

Organiser Professor C-H Chu
Level 6 **Credit value** 15 **Semester** A
Assessment 10% coursework, 90% final exam
Prerequisites MTH4101/MAS125 Calculus II, MTH5112/MAS212 Linear Algebra I. MTH5104/MAS111 Convergence and Continuity will be helpful.
Overlaps MAS346 Linear Operators and Differential Equations

Syllabus

1. Basic concepts: Discrete and continuous variables, scalar product, completeness. Linear operators, matrices, integral and differential operators. Adjoint operators, Hermitian and unitary operators. Eigenvectors/functions and eigenvalues of operators, degeneracy, completeness and examples. Eigenvalues/vectors for Hermitean and unitary operators.
2. Techniques: Existence and uniqueness of solutions of $Lf = g$. Inversion of operators, examples. Inverting a degenerate operator; applications to matrices and integral operators.
3. Ordinary differential equations: Sturm-Liouville operator. Green's function by expansion in eigenfunctions. Green's function obtained via method of variation of constant.
4. Partial differential equations: Solutions of homogeneous equation by separating variables. Legendre and Bessel functions. Solutions for inhomogeneous equations, Green's function for Laplacian by eigenfunction expansion.

Books
Other texts

- Mathews & Walker, Mathematical Methods of Physics (Benjamin)
- Friedman, Principles and Techniques of Applied Mathematics (Dover)
- Kreider/Kuller/Ostberg/Perkins, An Introduction to Linear Analysis (Addison-Wesley)
- Goertzel & Tralli, Some Mathematical Methods of Physics (McGraw-Hill)

MTH717U MSci Project

Organiser Dr J R Johnson
Level 7 **Credit value** 30 **Semester** A and B
Assessment Written report, presentation and (possibly) oral exam
Prerequisites You should begin preparation over the summer and before registering you must consult the module organiser. MTH5117/MAS237 Mathematical Writing useful.
Overlaps MAS410 MSci Project

Syllabus The project will be assessed primarily by a written report and, at the examiners' discretion, an oral examination, but also by a presentation of about 30 minutes in duration, to be given towards the end of semester 8. The contribution of the presentation will be on a sliding scale that will never decrease the project mark by more than 10% or increase it by more than 20%, and provided you make a reasonable attempt at giving a presentation it will not decrease your project mark.
Your report must present the study of some mathematical topic at fourth-year undergraduate level and must be your own work in the sense that it gives an original account of the material, but it need not contain new mathematical results. The length should be the equivalent of between 3,500 and 7,000 words. You can write your report in a single semester or spread over two semesters, depending on your other modules.

MTH6123 Mathematical Aspects of Cosmology

Organiser Dr A G Polnarev
Level 6 **Credit value** 15 **Semester** B
Assessment 10% coursework, 90% final exam
Prerequisites MTH5102/MAS204 Calculus III and MTH5106/MAS226 Dynamics of Physical Systems in Mathematical Sciences programme; PHY122 Mathematical Techniques 2 and PHY116 From Newton to Einstein in Physics programme.
Overlaps MAS347 Mathematical Aspects of Cosmology

Syllabus

1. Cosmography of the Universe: qualitative description of the contents of the Universe, including galaxies, large-scale structure, matter, radiation; cosmological principle, cosmic expansion and Hubble law.
2. Cosmic Microwave Background: its spectrum, anisotropy and polarisation.
3. Newtonian Cosmological Models: Derivation of evolution equations for scale factor within framework of Newtonian theory.

4. Relativistic Cosmological Models: Derivation of relativistic evolution equations (deceleration and Friedmann equations); determination of scale factor as function of time and key relationships between fundamental cosmological parameters.
5. A Brief History of the Universe: The age of the Universe; the dynamical role of matter, radiation, dark energy and curvature in the evolution of the scale factor.
6. Basic ideas of inflationary models and expansion with acceleration.
7. Mathematics of Observational Cosmology: Use of Robertson-Walker metric to study propagation of light-rays, and calculation of distance, surface areas and volumes; significance of particle horizon and cosmological red shift.
8. Origin of Large-scale Structure: Mechanism of gravitational instability; solutions of evolution equation for density perturbations in simple cosmological models.

Books
Reading list

- M Rowan-Robinson, Cosmology (OUP 3rd Edition)
- J Silk, The Big Bang (Freeman 2nd Edition)
- M Berry, Principles of Cosmology and Gravitation (CUP)
- J Islam, An Introduction to Mathematical Cosmology (CUP)
- B J Carr, Cosmology – old lecture notes

I24001 Mathematical Education for Physical and Mathematical Sciences

Organiser Dr Melissa Rodd at the Institute of Education; QM contact Dr F J Wright
Level 6 **Credit value** 15 **Semester** A and B
Assessment 50% coursework essay (to be submitted towards the end of Semester B) and 50% final exam (to be sat in May)
Prerequisites A second-year mean mark of at least 50%; also see below. Restricted to Queen Mary Mathematical Sciences students.

Syllabus The aim of this level-6 optional module is to introduce you to central ideas of mathematical education. It should be relevant to you if you are considering going into teaching after you graduate and it will also be relevant to you as a learner of mathematics. The module will be taught at the Institute of Education (IoE, http://www.ioe.ac.uk/) at 20, Bedford Way, seven minutes walk from Euston Square tube station. This module is valued by Queen Mary at 15 credits and will be counted as an MTH module for purposes of meeting study programme requirements.
Lectures will take place during both of Semesters A and B on Mondays from 5:00pm to 6:30pm for 20 sessions: ten in the autumn term and ten in the spring term, starting on 28th September 2009 and excluding reading weeks. When considering your timetable, you should allow 45 minutes travel time from Queen Mary to the Institute of Education. This means that you will not be able to attend any classes at Queen Mary after 4:00pm on Mondays. Individual tutorials will be arranged during Semester B to help with essay writing, and revision session(s) will be held late April / early May to help prepare for the exam.
To be allowed to register for this module you must:

- have a second-year mean mark of at least 50%;
- email the Director of Undergraduate Studies, Dr F. J. Wright, to express your interest, giving your full name and student number – if the module is oversubscribed then I will accept students in the order in which I receive their emails.

You should include this module on your Queen Mary module registration form by writing the module code I24001 in Section 2. This is the Queen Mary (intercollegiate) code for this module, although its IoE code is completely different. You will also need to obtain an intercollegiate module registration form from Queen Mary Registry, complete it, get it signed by the module organiser at the IoE, and return it to Dr F. J. Wright. (You do not need any other signatures from Queen Mary.) This form is necessary for your results to be transferred from the IoE to Queen Mary.

MTH736U Mathematical Statistics

Organiser Dr B Bogacka
Level 7 **Credit value** 15 **Semester** A
Assessment 100% final exam
Prerequisites MTH4106 Introduction to Statistics/MAS113 Fundamentals of Statistics I, MTH4101/MAS125 Calculus II, MTH5112/MAS212 Linear Algebra I
Overlaps MTH6136 Statistical Theory/MAS230 Fundamentals of Statistics II

Syllabus

- Probability Distribution Theory:
  - families of probability distributions;
  - functions of random variables;
  - conditional expectation;
  - common probability distributions;
  - multivariate random variables, marginal distributions, independence;
  - moment generating functions; correlation;
  - multivariate normal random variables;
  - distribution of quadratic forms;
  - Laws of Large Numbers and the Central Limit Theorem.
- Statistical Inference:
  - principles of data reduction including: sufficiency, likelihood, invariance;
  - point estimation including: properties of estimators (consistency, bias, minimum variance), Cramer-Rao lower bound, methods of estimation (method of moments, maximum likelihood, least squares), asymptotic properties;
  - interval estimation including: pivotal quantities, approximate maximum likelihood CIs, methods of evaluating interval estimators;
  - hypothesis testing including: statistical hypotheses, type I and II errors, power, Neyman-Pearson lemma, likelihood ratio tests, Wilks' theorem.

Books
Reading list

- J A Rice, Mathematical Statistics and Data Analysis (Duxbury)
- D Wackerley, W Mendenhall and R L Scheaffer, Mathematical Statistics with Applications (Duxbury)

MTH5117 Mathematical Writing

Organiser Professor F Vivaldi
Level 5 **Credit value** 15 **Semester** A
Assessment 20% coursework, 80% final exam
Prerequisites MTH4104/MAS117 Introduction to Algebra
Overlaps MAS237 Mathematical Writing, MAS233 Logic I: Mathematical Writing

Syllabus

1. Basic words and symbols of higher mathematics.
2. Mathematical notation: developing a coherent approach.
3. Describing the behaviour of functions.
4. Logical structures: the predicate algebra.
5. Basic proof techniques.
6. Existence statements.
7. Natural numbers: inductive arguments.
8. Definitions: what they are for and how to write them.
9. Intellectual property: giving credit, respecting copyright

Books
Main text

- F Vivaldi, Mathematical writing web-book, http://www.maths.qmul.ac.uk/~fv/books/mw/

Other texts

- G Chartrand, A Polymeny, and P Zhang, Mathematical proofs, a transition to advanced mathematics, Addison-Wesley (2003)
- D J Velleman, How to prove it: a structured approach, Cambridge University Press (1994)

MTH716U Measure Theory and Probability

Organiser Professor O Jenkinson
Level 7 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH5105/MAS221 Differential and Integral Analysis. MTH6126 Metric Spaces useful
Overlaps MAS409 Measure Theory and Probability

Syllabus This is an introductory course on the Lebesgue theory of measure and integral with application to Probability. Students are expected to know the theory of Riemann integration.

1. Measure in the line and plane, outer measure, measurable sets, Lebesgue measure, non-measurable sets.
2. Sigma-algebras, measures, probability measures, measurable functions, random variables.
3. Simple functions, Lebesgue integration, integration with respect to general measures. Expectation of random variables. Monotone and dominated convergence theorems, and applications.
4. Absolute continuity and singularity, Radon-Nikodym theorem, probability densities.
5. Possible further topics: product spaces, Fubini's theorem.

MTH6126 Metric Spaces

Organiser Professor M Jerrum
Level 6 **Credit value** 15 **Semester** A
Assessment 10% coursework, 90% final exam
Prerequisites MTH5104/MAS111 Convergence and Continuity

Syllabus

- Definition of metric space; examples, including finite metric spaces, function spaces, normed vector spaces, product spaces.
- Convergence and continuity in metric spaces.
- Equivalent metrics.
- Open and closed sets, properties, continuity in terms of pre-images of open sets.
- (Sequential) compactness, properties of compact spaces, uniform continuity. Bolzano-Weierstrass Theorem.
- Completeness; examples, including C[0,1]. Examples of completions of metric spaces. Contraction mappings, Banach fixed-point theorem, applications, e.g., to solutions of differential equations.
- Further topics if time allows, such as the Heine-Borel Theorem.

Books
Main texts

- W A Sutherland, Introduction to Metric and Topological Spaces, OUP 1975 (chapters 2, 5–9)
- Yu Safarov, Real Analysis II (lecture notes)

MTH6128 Number Theory

Organiser Professor P J Cameron
Level 6 **Credit value** 15 **Semester** B
Assessment 10% coursework, 90% final exam
Prerequisites MTH4104/MAS117 Introduction to Algebra
Overlaps MAS320 Number Theory

Syllabus

1. Continued fractions: finite and infinite continued fractions, approximation by rationals, order of approximation.
2. Continued fractions of quadratic surds: applications to the solution of Pell's equation and the sum of two squares.
3. Binary quadratic forms: equivalence, unimodular transformations, reduced form, class number. Use of continued fractions in the indefinite case.
4. Modular arithmetic: primitive roots, quadratic residues, Legendre symbol, quadratic reciprocity. Applications to quadratic forms.

Books
Main texts

- H Davenport, The Higher Arithmetic, Cambridge University Press (2008)
- Allenby & Redfern, Introduction to Number Theory with Computing, Edward Arnold (1989)

MTH6129 Oscillations, Waves and Patterns

Organiser Professor R K Tavakol
Level 6 **Credit value** 15 **Semester** B
Assessment 10% coursework, 90% final exam
Prerequisites MTH5102/MAS204 Calculus III, MTH5106/MAS226 Dynamics of Physical Systems
Overlaps MAS229 Oscillations, Waves and Patterns; PHY217 Vibrations and Waves

Syllabus

1. Oscillations: Review of restoring forces and SHM; damped oscillations, strong, weak and critical damping; forced damped oscillations, transient and steady state solutions; resonance.
2. Coupled oscillators: normal coordinates, normal modes of vibrations, derivation of wave equation as the limit of many coupled oscillators.
3. Waves: derivation of classical wave equation for string; D'Alembert's solution; travelling plane wave solutions; transverse vibrations on a string: harmonic waves, normal modes for string fixed at ends, solution by separation of variables; initial conditions and Fourier sine series; examples, such as vibrations and musical sounds.
4. Waves in fluids: linear surface waves on deep and shallow water; dispersion relation, phase and group velocities; waves on inclined beds, tsunamis.
5. Patterns: circular membranes (drums): modes of oscillation and their patterns; nonlinear waves and solitons; qualitative introduction to waves and pattern formation in other systems, e.g., biological and chemical systems.

Books
Other texts

- Vibrations and Waves: A P French (Chapman and Hall)
- The Physics of Vibrations and Waves: H J Pain (John Wiley and Sons)
- Vibrations and Waves in Physics: I G Main (Cambridge University Press)
- Wave Motion: Billingham and A C King (Cambridge University Press)
- Physics of Waves: W C Elmore and M A Heald (McGraw-Hill)
- Waves: C A Coulson (Oliver and Boyd)
- Wave Physics: S Nettel (Springer Verlag)

MTH4108 Probability I

Organiser Professor B Jackson
Level 4 **Credit value** 15 **Semester** A
Assessment 20% in-term tests, 80% final exam
Prerequisites A-level Mathematics or equivalent
Overlaps MAS108 Probability I

Syllabus

1. Probability: frequentist vs modelling vs subjective. Finite sample spaces (equiprobable or not); events as subsets. Sets, subsets, membership, set notation, union, intersection, complement, setminus. Commutative, distributive, and de Morgan's laws. Ordered and unordered pairs and higher products.
2. Functions, including domain, codomain, composition of functions, one-to-one, onto, bijections, inverse functions. Sequences: suffix notation, summation notation, change of suffix, manipulating sums.
3. Elementary ideas of probability theory; Kolmogorov axioms; additivity of probabilities of disjoint events. Sigma notation with suffix i. Simple proofs from the axioms. Inclusion-exclusion. Propositions, logical operations, negation, and, or, converse, equivalent, ideas of proof.
4. Sampling with and without replacement. Counting. Binomial Theorem.
5. Independent events: definition, examples. Multiplication law. Three or more events.
6. Conditional probability. Definition. Sampling without replacement done in stages rather than as set of outcomes. Proof by induction that $P(E_1 \cap E_2 \cap \cdots \cap E_n) = P(E_1) \times P(E_2 \mid E_1) \times \cdots \times P(E_n \mid E_1 \cap \cdots \cap E_{n-1})$. Theorem of Total Probability. Use sigma notation with suffix i.
7. Bayes' Theorem and its use to calculate 'inverse' probabilities like conditional probability of having disease D given that test for D is positive. Discrete random variables as functions from sample space to $\mathbb{R}$.
8. Probability mass function, mean. Variance. Mean and variance of $aX + b$.
9. Familiarity with the following distributions (including pmf, mean, variance, what they are used to model): Bernoulli, binomial, geometric, hypergeometric, Poisson. Cumulative distribution function for discrete random variables. Informal introduction to continuous random variables. Cumulative distribution function, probability density function. Mean, variance. $E(g(X))$. Median and quartiles.
10. Familiarity with exponential and uniform distributions. Monotone 1-dimensional transformations of random variables. Proof of pdf of new random variables in continuous case.
11. Joint distribution of two random variables in some simple discrete cases. Marginal distributions. Independent random variables. Covariance and correlation. Independence implies zero covariance. Mean and variance of $aX + bY$ in general. Hence re-derive mean and variance of binomial as sum of independent Bernoulli's. Conditional distribution in simple discrete cases. Conditional random variables. Theorem of Total Probability for expectation. Hence re-derive mean and variance of geometric distribution.

Books
Main texts
- J L Devore, Probability and Statistics (Thomson Brooks/Cole, 6th Edition)
- Lindley/Scott, New Cambridge Statistical Tables (CUP)

Other texts
- Hines/Montgomery, Probability & Statistics in Engineering & Management Science (Wiley)
- J A Rice, Mathematical Statistics & Data Analysis (Wadsworth)
- S Ross, A First Course in Probability (Prentice Hall)

MTH5118 Probability II
Organiser Professor I Goldsheid
Level 5 **Credit value** 15 **Semester** A
Assessment 10% coursework, 10% in-term test, 80% final exam
Prerequisites MTH4101/MAS125 Calculus II, MTH4106/MAS113 Introduction to Statistics.
Overlaps MAS228 Probability II

Syllabus
Part A. Discrete probability
1. Probability generating function and its use (factorial moments, sums of independent random variables).
2. Revision of conditional probability. Computing probabilities by conditioning. The gambler's ruin problem.
3. Conditional expectation. Computing expectations by conditioning. Conditional variance. Expected value and variance of a random number of random variables.

4. Branching processes via probability generating function.

Part B. Continuous probability

1. Joint distributions. Computing probabilities from the joint probability density function. Uniform distribution. Marginal distributions. Expectation of a function of random variables (without proof). Covariance and correlation coefficient. Independence for two random variables. Independence in the multivariate setting.
2. Transformation of random variables (technique and simple examples of its use). t- and F-distributions.
3. Moment generating function and its use. Sums of independent random variables. Gamma distribution. Chi-squared distribution.
4. Bivariate normal distribution (definition and basic properties). Multivariate normal distribution in matrix notation.
5. Conditioning on a continuous random variable. Conditional expectation. Computing expectations by conditioning.

Part C. Limit theorems

1. Chebyshev's inequality. The weak law of large numbers.
2. Central limit theorem (by the way of moment generating function).

Books
Main text

- S Ross: A First Course in Probability

MTH6130 Probability III

Organiser Dr J R Johnson
Level 6 **Credit value** 15 **Semester** B
Assessment 10% coursework, 90% final exam
Prerequisites MTH5118/MAS228 Probability II
Overlaps MAS338 Probability III

Syllabus

1. Discrete Markov chains (general formalism). Markov chain models.
2. Markov chains with absorbing states (probability of absorption in a given state, expected time to absorption).
3. Random walks on a finite interval with absorbing and reflecting boundaries. Random walks on an infinite interval (probability of escaping to infinity, probability of return). Recurrent and transient random walks.
4. Long run behaviour of Markov chains.
5. Poisson distribution as the law of rare events. Definition and basic properties of the Poisson process. Waiting and sojourn times. Relation to the uniform distribution. Computing expectations of additive functionals of waiting times.
6. Birth and death processes. Queueing systems.

The following may be included if time permits: Renewal processes and/or Brownian motion.

Books
Main text

- N M Taylor and S Karlin, An Introduction to Stochastic Modeling

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MTH6132 Relativity

Organiser Professor R K Tavakol
Level 6 **Credit value** 15 **Semester** A
Assessment 10% coursework, 90% final exam
Prerequisites MTH4102/MAS118 Differential Equations, MTH4101/MAS125 Calculus II, MTH5112/MAS212 Linear Algebra I
Overlaps MAS322 Relativity

Syllabus

1. Special Relativity: Newtonian mechanics and Galilean relativity. Maxwell's equations and special relativity. Lorentz transformations and Minkowski space-time. Clocks and rods in relative motion.
2. Vectors in Special Relativity: 4-vectors and the Lorentz transformation matrix 4-velocity, 4-momentum, 4-acceleration. Relativistic dynamics and collisions. Optics: redshift and aberration
3. Tensors in special relativity: Metrics and forms. Tensors and tensor derivatives. Stress-energy tensor. Perfect fluids.
4. Conceptual Basis of General Relativity: Problems with Newtonian gravity. Equivalence principle.
5. Curved Space-time and General Relativity: Tensor calculus. Covariant derivatives and connections. Parallel transport and geodesics. Curvature and geodesic deviation. Einstein's field equations.
6. Application of General Relativity: Schwarzschild solution. Tests of general relativity. Black holes and gravitational collapse.

Books
Main texts

- M V Berry, Principle of Cosmology and Gravitation (CUP) [Elementary]
- R d'Inverno, Introducing Einstein's Relativity (Clarendon Press, Oxford) [Intermediate]
- B F Schutz, A First Course in General Relativity (CUP) [Intermediate]
- W Rindler, Essential Relativity: Special, General and Cosmological (Springer-Verlag) [Intermediate]
- A Einstein, The Principle of Relativity (Dover) [Classical]

MTH720U Relativity and Gravitation

Organiser Dr A G Polnarev
Level 7 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH6132/MAS322 Relativity
Overlaps MAS412 Relativity and Gravitation

Syllabus

- Introduction to general relativity.
- Derivation from basic principles of the Schwarzschild, Reisner-Nordstrom, Kerr and Kerr-Neuman solutions of Einstein's field equations.
- Physical aspects of strong gravitational fields around black holes.
- Generation, propagation and detection of gravitational waves.
- Weak general relativistic effects in the solar system and binary pulsars.
- Alternative theories of gravity and experimental tests of general relativity.

MTH722U Rings and Modules

Organiser Dr M Fayers
Level 7 **Credit value** 15 **Semester** A
Assessment 100% final exam
Prerequisites MTH5112/MAS212 Linear Algebra I, MTH6104/MAS305 Algebraic Structures II
Overlaps MAS427 Rings and Modules

Syllabus This course is designed to give an overview of rings and modules. The definition of a ring will not be assumed, but familiarity with the basic definitions will be helpful. What is essential is a thorough grounding in linear algebra.

1. Definitions of rings, modules, homomorphisms. The Isomorphism Theorems.
2. Inverses and division rings. Wedderburn's theorem on finite division rings.
3. Direct sums of rings and modules; central idempotents.

4. Semisimple modules. The Artin-Wedderburn Theorem on semisimple rings.
5. Chain conditions, Artinian and Noetherian rings and modules. Composition series.
6. The Jacobson radical and Artinian rings. Hopkins's Theorem.

Books
Recommended reading

- J A Beachy, Introductory lectures on rings and modules, CUP 1999
- P M Cohn, Introduction to ring theory, Springer 2000

The following general algebra texts may be useful for consultation:

- P M Cohn, Algebra (3 vols), Wiley 1974–77
- N Jacobson, Basic Algebra (2 vols) Freeman 1980

MTH5119 Sampling, Surveys and Simulation

Organiser Dr R A Sugden
Level 5 **Credit value** 15 **Semester** A
Assessment 10% in-term test, 24% questionnaire design, 16% coursework, 50% final exam
Prerequisites MTH4106 Introduction to Statistics/MAS113 Fundamentals of Statistics I, MTH4101/MAS125 Calculus II
Overlaps MAS234 Sampling, Surveys and Simulation

Syllabus The techniques covered will be applied to data from various areas of business, economics, science and industry.

1. Simple, cluster and stratified random sampling – how and why they arise, estimation in infinite population models, finite population corrections.
2. Questionnaire / survey design – length and layout of questionnaire, piloting, confidentiality and ethical issues, question content and wording, questionnaire flow, surveys without questionnaires.
3. Simulation – how to sample from different distributions, simulation of simple stochastic processes, illustrations of theoretical results (sampling distributions, laws of large numbers, central limit theorem).

Books
Main text

- V Barnett: Sample Survey Principles and Methods, 3rd edition (Arnold 2002)

Other text

- W G Cochran: Sampling Techniques (Wiley, 1977)

MTH724U Solar System

Organiser Professor C D Murray
Level 7 **Credit value** 15 **Semester** A
Assessment 100% final exam (up to 10% of final mark can be obtained from coursework)
Prerequisites MTH5106/MAS226 Dynamics of Physical Systems
Overlaps MAS423 Solar System

Syllabus The material presented in this module will be chosen from the following:

1. General overview/survey
2. Fundamentals: two-body problem, continuum equations
3. Terrestrial planets: interiors, atmospheres
4. Giant planets: interiors, atmospheres
5. Satellites: three-body problem, tides
6. Resonances and rings
7. Solar nebula and planet formation
8. Asteroids, comets and impacts

Books
Main texts

- I de Peter & J J Lissauer, Planetary Sciences (Cambridge University Press)
- C D Murray & S F Dermott, Solar System Dynamics (Cambridge)

Other texts

- B Bertotti, P Farinella & D Vokrouhlicky, Physics of the Solar System (Kluwer Academic Publishers)
- J S Lewis, Physics and Chemistry of the Solar System (2nd edition, Elsevier Academic Press)
- J K Beatty, C C Petersen& A Chaikin, The New solar System (4th edition, Cambridge University Press, Sky Publishing)

MTH5120 Statistical Modelling I

Organiser Dr L Pettit
Level 5 **Credit value** 15 **Semester** B
Assessment 20% coursework, including any in-term tests, 80% final exam
Prerequisites MTH5112/MAS212 Linear Algebra I, MTH5118/MAS228 Probability II
Overlaps MAS232 Statistical Modelling I

Syllabus The techniques covered will be applied to data from various areas of business, economics, science and industry.

1. Relationships among variables and basic concepts of statistical modelling, response and explanatory variables.
2. The Normal-linear model: definition, matrix form, simple, multiple and polynomial regression models.
3. Matrix algebra: trace, transpose and inverse of square matrices, manipulation of matrix equations, vector differentiation.
4. Estimation: maximum likelihood, least squares, Gauss-Markov Theorem, properties of estimators, estimating mean responses, estimating σ^2.
5. Assessing fitted models: analysis of variance, $\mathbb{R}^2$, lack of fit, residuals and model checking, outliers.
6. Model selection: transformation of the response variable, order of polynomial models, variable selection.
7. Inference: confidence intervals for parameters and mean response, testing for parameters and mean response.
8. Uses of linear models – prediction, control, optimisation.
9. Problems: leverage and influence, multicollinearity.
10. Use of Minitab to apply the theory to practical data analysis.

Books
Main text

- B Abraham and J Ledolter, Introduction to Regression Modeling (Duxbury)
- Lindley/Scott, New Cambridge Elementary Statistical Tables (CUP)

Other texts

- Draper & Smith, Applied Regression Analysis (Wiley)
- Sen & Srivastava, Regression Analysis (Springer)

MTH6134 Statistical Modelling II

Organiser Dr H Grossmann
Level 6 **Credit value** 15 **Semester** A
Assessment 20% coursework, 80% final exam
Prerequisites MTH5120/MAS232 Statistical Modelling I
Overlaps MAS339 Statistical Modelling II

Syllabus Extended use of the comprehensive statistical package GenStat is developed as it is required in the module. The methods introduced are applied to data from various applications in business, economics, science and industry.

1. Qualitative explanatory variables – models, factors, main effects and interactions.

2. Indicator variables – representation as linear regression models.
3. Parameterisations and constraints – intrinsic and extrinsic aliasing.
4. Vector spaces and least squares estimation using projections.
5. Nested, crossed and general structures.
6. Random effects – variance components, mixed models.

Books
Main text
- Krzanowski, W J (1998). An Introduction to Statistical Modelling. Arnold

Other texts
- Draper, N R and Smith, H (1998). Applied Regression Analysis, 3rd edition. Wiley
- Lindley, D V and Scott, W F (1995). New Cambridge Statistical Tables, 2nd edition. Cambridge University Press
- Montgomery, D C (1997). Design and Analysis of Experiments, 4th edition. Wiley
- Seber, G A F (1980). The Linear Hypothesis: A General Theory, 2nd edition. Griffin

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MTH6136 Statistical Theory
Organiser Dr R A Sugden
Level 6 **Credit value** 15 **Semester** B
Assessment 10% in-term, 90% final exam
Prerequisites MTH5118/MAS228 Probability II
Overlaps MAS230 Fundamentals of Statistics II

Syllabus The theory developed will be used to justify the methods introduced in MTH4106 Introduction to Statistics or MAS113 Fundamentals of Statistics I and will be used to analyse data from a variety of applications.

1. Estimation: bias, sufficiency, Cramer-Rao lower bound, minimum variance unbiased estimators.
2. Methods of estimation: method of moments, maximum likelihood, least squares, properties of estimators obtained from these methods, asymptotic properties of MLEs.
3. Confidence intervals: methods of obtaining CIs using pivots, likelihood CIs.
4. Testing: power, simple and composite hypotheses, Neyman-Pearson Lemma, uniformly most powerful tests, likelihood ratio tests, Wilks' Theorem.

Books
Main text
- Wackerly, D D, Mendenhall, W and Scheaffer, R L (2002). Mathematical Statistics with Applications, 6th edition. Duxbury

Other texts
- Hogg, R V and Tanis, E A (2006). Probability and Statistical Inference, 7th edition. Prentice Hall
- Larson, H J (1982). Introduction to Probability Theory and Statistical Inference, 3rd edition. Wiley
- Lindley, D V and Scott, W F (1995). New Cambridge Statistical Tables, 2nd edition. Cambridge University Press
- Miller, I and Miller, M (2004). John E Freund's Mathematical Statistics with Applications, 7th edition. Prentice Hall

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MTH725U Stellar Structure and Evolution
Organiser Dr S V Vorontsov
Level 7 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH5102/MAS204 Calculus III, MTH6129/MAS229 Oscillations, Waves and Patterns, or equivalent
Overlaps MAS415 Stellar Structure and Evolution

Syllabus

1. Observational properties of stars, the H-R diagram, binary stars, clusters, solar and stellar oscillations.
2. Physical properties of stellar interiors: virial theorem, gravitational energy, radiative transfer, opacity and equation of state, convection, nuclear reactions.
3. Equations of stellar structure and evolution. Order of magnitude estimates, simple stellar models. Convective cores and envelopes, the Cowling model, models of the Sun, acoustic models, massive stars, small mass stars.
4. Pre-main sequence evolution, evolution on the main sequence, post main sequence evolution. Degeneracy, models of white dwarfs, models of red giants. Late stages of stellar evolution, nucleosynthesis.
5. Selected topics from: the solar wind, mass loss, stellar rotation, binary stars, convective overshooting, mixing and diffusion. Solar neutrino problem, nuclear astrophysics.

The module includes some exposure to simple numerical techniques of stellar structure and evolution; computer codes in Fortran.

MTH726U The Galaxy

Organiser Dr W J Sutherland
Level 7 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH5102/MAS204 Calculus III
Overlaps MAS430 The Galaxy

Syllabus

- Introduction: galaxy types, descriptive formation and dynamics.
- Stellar dynamics: virial theorem, dynamical and relaxation times, collisionless Boltzmann equation, orbits, simple distribution functions, Jeans equations.
- The interstellar medium: emission processes from gas and dust (qualitative only), models for chemical enrichment.
- Dark matter – rotation curves: bulge, disk, and halo contributions.
- Dark matter – gravitational lensing: basic lensing theory, microlensing optical depth.
- The Milky Way: mass via the timing argument, solar neighbourhood kinematics, the bulge, the Sgr dwarf.

MTH6138 Third-Year Project

Organiser Dr J R Johnson
Level 6 **Credit value** 15 **Semester** A or B
Assessment Written report, presentation and (possibly) oral exam
Prerequisites Before registering you must consult the module organiser. MTH5117/MAS237 Mathematical Writing useful.
Overlaps MAS342 Third-Year Project. You will not normally be allowed to take this option together with another project module.

Syllabus A project in any area of mathematics, including astronomy and computing, which is offered in both of semesters 5 and 6. It may be a simplified version of an MSci project, although some MSci projects may not be available as third-year projects. The list of available MSci projects and supervisors is available on the School of Mathematical Sciences website. The project will be assessed primarily by a written report and, at the examiners' discretion, an oral examination, but also by a presentation of about 20 minutes in duration, to be given towards the end of semester 6. The contribution of the presentation will be on a sliding scale that will never decrease the project mark by more than 10% or increase it by more than 20%, and provided you make a reasonable attempt at giving a presentation it will not decrease your project mark.
Your report must present the study of some mathematical topic at third-year undergraduate level and must be your own work in the sense that it gives an original account of the material, but it need not contain new mathematical results. The length should be the equivalent of between 3,000 and 4,000 words.

MTH6139 Time Series

Organiser Dr D S Coad
Level 6 **Credit value** 15 **Semester** A
Assessment 20% in-term, 80% final exam
Prerequisites MTH5118/MAS228 Probability II, MTH5120/MAS232 Statistical Modelling I
Overlaps MAS328 Time Series, ECN323 Economic Forecasting

Syllabus The course includes time series analysis using MINITAB. The methods developed are applied to data arising in applications in economics, business, science and industry.

1. General introduction and motivation.
2. Trends and seasonality and their removal by moving averages. Differencing.
3. Review of probability.
4. Time series as a stationary stochastic process.
5. Modelling of time series in the time domain. Development of AR(p) and MA(q) models in general and their detailed study for the case of $p = q = 1$.
6. ARMA models.
7. Model identification using the ACF and PACF.
8. Estimation of parameters by moments, least squares and maximum likelihood methods.
9. Forecasting by least squares and conditional expectations.
10. ARIMA models.

Books
Main texts

- P J Brockwell and R A Davis, An Introduction to Time Series and Forecasting (Springer)
- C Chatfield, The Analysis of Time Series, an Introduction (Chapman & Hall)

Other texts

- R Shumway & D Stoffer, Time series Analysis and Its Applications (Springer)
- P J Brockwell & R A Davis, Time Series: Theory and Methods (Springer)
- P Diggle, Time Series: A Biostatistical Introduction (Oxford)
- A C Harvey, Time Series Models (Philip Allan)

MTH734U Topics in Probability and Stochastic Processes

Organiser Dr R A Sugden
Level 7 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH5118/MAS228 Probability II. This module is suitable for third-year undergraduates.
Overlaps MAS420 Topics in Probability and Stochastic Processes

Syllabus Topics will be chosen from the following list:

1. Borel-Cantelli lemma, Kolmogorov's inequalities, strong law of large numbers.
2. Weak convergence of distributions. The Central Limit Theorem.
3. Recurrent events and renewal theory.
4. Further topics in random walks.
5. General theory of Markov chains. Classification of states and ergodic properties.
6. Continuous time Markov Processes.

Books
Main texts

- W Feller, An Introduction to Probability Theory and its Applications I (Wiley)
- H M Taylor and S Karlin, An Introduction to Stochastic Modeling, 3rd Edition (Academic Press)

MTH732U Topology

Organiser Dr M Walters
Level 7 **Credit value** 15 **Semester** B
Assessment 100% final exam
Prerequisites MTH4104/MAS117 Introduction to Algebra, MTH6126 Metric Spaces. MTH5100/MAS201 Algebraic Structures I useful
Overlaps MAS329 Topology

Syllabus

1. Topological spaces: examples including discrete, indiscrete, metric and co-finite topologies.
2. Continuity and convergence, homeomorphisms, topological and non-topological properties.
3. Paths and path connectedness.
4. Compactness in a topological space, Heine-Borel theorem, compact implies sequentially compact in metric spaces. Statement of converse.
5. New spaces: subspaces, product spaces, identification spaces (especially of a square).
6. Paths and path homotopies. Simply connectedness.
7. The fundamental group, definition and elementary properties. Fundamental group of a circle. Path and homotopy lifting (proofs non-examinable).
8. Brouwer Fixed Point Theorem in two dimensions. Borsuk-Ulam Theorem in two dimensions.
9. Fundamental group of identifications of the square including the torus and Klein bottle.

Books
Main texts

- B Mendelson, Introduction to topology (Dover Publications)
- W A Sutherland, Introduction to metric and topological spaces (CUP)