Week 10

1 Post-main sequence evolution of high mass stars

Stars with initial masses greater than about 8 $M_\odot$ are expected to evolve through all the stages of nuclear burning. The process begins with hydrogen burning at about 2×10^7 K, and proceeds at progressively higher temperatures through helium, carbon, neon, oxygen and silicon burning. Silicon burning at about 3×10^9 K leads to a star with a central core of iron, surrounded by concentric shells containing silicon, oxygen, neon, carbon, helium and hydrogen. Because energy cannot be released by the thermonuclear fusion of iron (the most stable form of nuclear matter consists of nuclei near ^{56}Fe in the periodic table), the central core contracts. Initially, this contraction can be controlled by the pressure of the dense gas of degenerate electrons in the core, but as the silicon burning in the surrounding shell deposits more iron onto the central core, the degenerate electrons in the core become increasingly relativistic. When the core mass reaches 1.4 $M_\odot$, the electrons become ultra-relativistic and they are no longer able to support the core.

1.1 Core-collapse supernova explosions

Once the innermost region of stellar core approaches 1.4 $M_\odot$, the core is on the brink of a catastrophe, and what follows is an uncontrolled collapse of the stellar core.

To understand the onset of the collapse, we note that when a body contracts under gravity, gravitational energy is converted into internal energy and the temperature rises. If this leads to the activation of exothermic nuclear fusion, the internal energy increases, the pressure rises, and the contraction is opposed. The opposite happens if an energy-absorbing (endothermic) process is activated: energy is absorbed, the effectiveness of the pressure is diminished, and slow gravitational contraction turns into rapid gravitational collapse.

There are two energy absorbing processes which could drive the iron core of a star into an uncontrolled collapse. They are the photodissociation of atomic nuclei and the capture of electrons via inverse beta decay.

1.1.1 Nuclear photodissociation

As the core collapses, the temperature rises, and photons become energetic enough to initiate nuclear photodissociation. For simplicity, assume that we have the following reaction (in reality photodissociation leads to the formation of different species of nuclei)

$$\gamma +^{56}\mathrm{Fe} \rightleftharpoons 13\,^4\mathrm{He} + 4n, \tag{1}$$

and we see that photon energy is used to unbind atomic nuclei, leading to a net loss of thermal energy in the core that supports against gravity. If we label the particles according to their mass number, then we have

$$Q = (13m_4 + 4m_1 - m_{56})c^2 = 124.4\,\mathrm{MeV}, \tag{2}$$

i.e. 1 kg of ^{56}Fe absorbs 2×10^{14} J of energy - equivalent to 50 kilotons of TNT. Hence, the total amount of energy that can be absorbed by this process, assuming that we have an iron core of mass ≈ 1.4 $M_\odot$, is approximately

$$2 \times 10^{14}\,\mathrm{J} \times 1.4 \times 2 \times 10^{30} \approx 6 \times 10^{44}\,\mathrm{J}.$$

This is equivalent to the energy radiated by the Sun over 10^{10} years.

1.1.2 Electron capture

Neutrons are normally unstable to beta decay with a half-life of 10.25 minutes

$$\mathrm{n} \rightarrow \mathrm{p} + e^- + \bar{\nu}_e.$$

The combined energy of the electron and anti-neutrino is 1.3 MeV (i.e. the mass-energy difference between the neutron and proton). Hence, electrons with energies up to 1.3 MeV are produced. This process of neutron beta decay can be switched off if the neutrons are immersed in a dense gas of degenerate electrons, and all of the energy states with energy up to 1.3MeV are fully occupied. We recall from week 9 that the Fermi momentum is given by

$$p_0 = h\left(\frac{3n_e}{8\pi}\right)^{1/3}, \tag{3}$$

and the associated electron energy is

$$E_0^2 = p_0^2c^2 + m_e^2c^4. \tag{4}$$

If the gas has a density that corresponds to a larger Fermi momentum than this, then electrons will be present with energy > 1.3 MeV, and these can be captured by protons to form neutrons via inverse beta decay

$$e^- + \mathrm{p} \rightarrow \mathrm{n} + \nu_e, \tag{5}$$

a process called *neutronisation*. Protons in the core of evolved massive stars are bound up in nuclei, but can still capture electrons in reactions like

$$e^- +^{56}\mathrm{Fe} \rightarrow^{56}\mathrm{Mn}. \tag{6}$$

This occurs when the density exceeds 1.1×10^{12} kg m^{-3}, for which the Fermi energy $E_0 = \sqrt{p^2c^2 + m_e^2c^4} =$ 3.7 MeV, the value needed for inverse beta decay of ^{56}Fe to occur. Normally, a ^{56}Mn nucleus decays to ^{56}Fe with a half-life of 2.6 hours, but in a stellar core it captures an electron to form ^{56}Cr. Electron capture by inverse beta decay on nuclei in a stellar core becomes very rapid when $\rho \approx 10^{14}$ kg m^{-3}. The kinetic energy of degenerate electrons is converted into the kinetic energy of electron neutrinos, ν_e, which escape from the core. These energy absorbing processes are so effective that the collapse of the core is almost unopposed by pressure effects, and the core can collapse almost freely under gravity on a free-fall time scale (derived in week 1)

$$\tau_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32G\rho}}, \tag{7}$$

which is on the order of 1 millisecond for $\rho \approx 10^{12}$ kg m^{-3}. The total amount of energy lost may be estimated as follows. An ^{56}Fe core with 1.4 M$_\odot$ has $\approx 10^{57}$ electrons, which can give rise to 10^{57} ν_e. The average energy of the captured electrons is ≈ 10 MeV when $\rho \approx 2\times10^{13}$ kg m^{-3}, hence the energy lost (expressed in Joules) is

$$E_{\mathrm{cap}} \approx 10^{57} \times (10 \times 1.6 \times 10^{-13}) = 1.6 \times 10^{45}\ \mathrm{J}.$$

Hence, a key expectation is that the collapse of an iron stellar core will be accompanied by the emission of a very large flux of neutrinos.

The collapse is rapid and almost unopposed until a density comparable to the density of nuclear matter is reached. The nuclear forces, which are attractive over scales $\sim 10^{-15}$ m and repulsive on smaller scales, and neutron degeneracy, are expected to resist further compression and bring the collapse to a fairly sudden halt. Using the fact that the radius of a nucleus is given by

$$R = r_0A^{1/3}, \qquad r_0 \approx 1.2\times10^{-15}\ \mathrm{m},$$

where A is the atomic mass number, we find that the density of nuclear matter

$$\rho_{\mathrm{nuc}} \approx \frac{3Am_{\mathrm{n}}}{4\pi R^3} = \frac{3m_{\mathrm{n}}}{4\pi r_0^3} = 2.3\times10^{17}\ \mathrm{kg\ m^{-3}}.$$

Upon exceeding this density, the core is expected to rebound strongly, and sets up a shock wave that propagates outwards and travels through the material that is falling towards the centre. Theoretical calculations and sophisticated computer simulations suggest that this shock is able to reverse the inwards fall of stellar material surrounding the core and produce an outward expulsion, a *supernova.*

Supernovae are very energetic explosions: the observed kinetic energy of the debris is typically about 10^{44} J, and the energy output in the optical is about 10^{42} J. These numbers, however, should be compared with the gravitational binding energy that is released by the collapse of the core

$$\Delta\Omega \approx -\int_{R_{\rm initial}}^{R_{\rm final}} \frac{GM^2}{r^2} dr \approx \frac{GM^2}{R_{\rm final}} = 3\times 10^{46}\left(\frac{M}{M_\odot}\right)^2\left(\frac{10 \text{ km}}{R}\right) \text{ J},$$

assuming that $R_{\rm final} \ll R_{\rm initial}$. Before the collapse $R_{\rm initial} \approx 1000$ km, and afterwards we see that $R_{\rm final} \approx 10$ km (as will be proved below).

Obviously the gravitational energy released is very large compared to what we observe in the kinetic energy and photon energy associated with a supernova explosion, so it is clear that the energy required to drive the explosion is available. The question is: what is the actual energy loss channel that explains the discrepancy between the observed energy release and the change in gravitational potential energy? (since these must balance to conserve the total energy). The answer is that the collapse leads to the formation of a hot, bloated neutron star, and the energy density in this object is large enough that spontaneous production of neutrino–anti-neutrino pairs occurs, and these can more or less freely escape from the collapsed core, leading to most of the gravitational binding energy being released through a burst of neutrino emission. This theoretical expectation was beautifully confirmed by detection of the neutrino burst from the supernova 1987A by the Kamiokande (Japan) and Irvine-Michigan-Brookhaven (USA) neutrino experiments, as shown in figure 1.

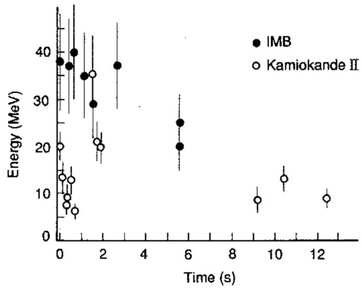

Figure 1:

Among the important by-products of a supernova is that the mixture of products of thermonuclear reactions accumulated around the core is ejected into the interstellar medium, and hence enriches it with heavy elements.

The collapse of the iron core of a massive star is the cause of the so-called Type II supernovae. The collapse is expected to leave a core residue, either a *neutron star*, or an overweight neutron star that collapses to form a *black hole.* Type Ia supernovae, which are used as standard candles for measuring distances on cosmological scales, are caused by the thermonuclear detonation of a carbon-oxygen white dwarf, which increases its mass by accreting material from a close companion binary star, or by coalescing with another white dwarf star binary companion after they spiral-in due to emission of gravitational waves. If such a white dwarf exceeds the Chandrasekhar limit, then it will contract, heat up, and ignite an uncontrolled thermonuclear explosion which destroys the star completely. This arises because the effective thermostat that operates in main sequence stars to maintain a constant rate of nuclear energy generation cannot operate in a star whose support against gravity is dominated by degeneracy pressure instead of thermal pressure.

1.2 Neutron stars

A neutron star is born as a hot residue of the collapsed core of a massive star. The typical internal temperature is initially between 10^{11} and 10^{12} K. It rapidly cools by neutrino emission, and is expected to reach a temperature on the order of 10^9 K in a day and 10^8 K in 100 years. These are high temperatures according to terrestrial and solar standards, but they are low when compared to the standards set by the high densities of matter inside a neutron star. The electrons, photons and above all the neutrons, which appear to be the dominant constituents of neutron stars, are degenerate and occupy the lowest possible states consistent with Pauli's exclusion principle. The characteristic radius of a neutron star is about 15 km, which is about 2000 times smaller than the typical size of a white dwarf given by eqn. (26) in the week 9 lecture notes.

We now discuss the nature of the matter inside a neutron star, to understand why neutrons are likely to be the main constituents. Normally, the most stable nuceli are near ^{56}Fe in the periodic table. Less massive nuclei are less stable because there is a higher fraction of nuclei near the surface.

More massive nuclei are less stable because the Coulomb repulsion starts to become more important. This balance changes in the presence of degenerate electrons, and as discussed above once the Fermi energy is high enough energetic electrons can cause $p \rightarrow n$ through inverse beta decay. The associated neutronisation of nuclei leads to the formation of neutron-heavy isotopes such as ^{78}Ni and ^{76}Fe that are the most stable nuclei in a degenerate electron gas when $\rho \approx 10^{14}$ kg m^{-3}. At densities $\rho > 4 \times 10^{14}$ kg m^{-3}, the phenomenon of neutron drip occurs, and neutrons start to dissociate from nuclei. The result is a dense gas in which electrons, neutrons and nuclei co-exist. The equation of state for such a mixture is fairly well understood for such a mixture for $\rho < \rho_{\rm nuc} = 2.3 \times 10^{17}$ kg m^{-3}, but at higher densities the nuclei start to merge with another, and the state of matter is now a dense gas consisting of electrons, protons and neutrons. The equation of state now becomes very complicated to understand and calculate, since it depends not just on neutron degeneracy (so adopting an equation of state similar to that used for white dwarfs would not be accurate), but also on the complex short range interactions among the nucleons.

To understand why neutrons are the dominant constituent, let us neglect the mutual interactions between the neutrons and protons and just consider a degenerate gas of electrons, protons and neutrons. As discussed above, neutrons are prevented from decaying because the beta decay of neutrons

$$n \rightarrow p + e^- + \bar{\nu}_e$$

is blocked due to the electron degeneracy, whereas the degeneracy ensures a plentiful supply of energetic electrons that cause the inverse beta decay of protons

$$p + e^- \rightarrow n + \nu_e,$$

hence converting protons into neutrons.

1.2.1 Sizes of neutron stars

Given that we have demonstrated that neutron stars are expected to be composed primarily of neutrons, we can say that the number density of neutrons is

$$n_{\rm n} \approx \frac{\rho}{m_{\rm n}},$$

where $m_{\rm n}$ is the mass of the neutron (almost the same as $m_{\rm H}$). If we make the simplifying assumption that a neutron star is supported entirely by a non-relativistic, fully-degenerate neutron gas, then we can adopt the analysis used in week 9 to obtain the mass radius relation for a white dwarf star, noting that the equation of state is the same as that for an $n = 3/2$ polytrope. For a 1.5 $M_\odot$ star, this predicts the radius to be $R = 13$ km.

1.2.2 Maximum mass of a neutron star

To a first approximation, neutrons play the same supporting role in a neutron star as electrons in a white dwarf. As discussed in the lecture notes of week 9, the smaller momenta of electrons compared to protons and neutrons in a gas of a given temperature means that electron degeneracy pressure sets in at lower densities than neutron degeneracy pressure, since we require that the Fermi momentum given by

$$p_0 = \frac{h}{2}\left(\frac{3n_e}{\pi}\right)^{1/3} \tag{8}$$

is larger than that associated with the thermal motion of the particles before degeneracy pressure becomes important. Just as degenerate electrons can fail to provide enough support to prevent the collapse of a white dwarf above a certain mass, the Chandrasekhar limit, degenerate neutrons are unable to support a neutron star with a mass that exceeds a certain value.

The physics underlying the Chandrasekhar limit is clear cut. As the mass of the white dwarf approaches the limit, the central density increases and the degenerate electrons become increasingly degenerate. At the Chandrasekhar limit the electrons are ultra-relativistic and the star collapses.

A similar phenomenon involving neutrons is expected in a neutron star, but there are a number of important differences. First, the interactions between the neutrons are important at the high densities found in a neutron star. Second, the gravitational fields are very strong, and Einstein's theory of gravity, not Newton's, should be used to describe the equilibrium structure of a neutron star. However, these differences do not alter the fundamental result that there is a maximum mass for a neutron star. Their main effect is to make the calculation of the maximum mass of a neutron star very difficult.

We can make a simple estimate by adopting the same analysis as used in week 9 for white dwarfs, and assuming that as the mass of a neutron star increases, its neutrons become ultra-relativistic, and the equation of state changes from that of a $n = 3/2$ polytrope to that of an $n = 3$ polytrope. A self-gravitating body with such an equation of state sits on the stability/instability boundary, and has a mass that is determined entirely by the polytropic constant K. In this case, the polytropic constant is given by fundamental constants as described by eqn. (15) in the week 9 lecture notes:

$$K_2 = \frac{hc}{8}\left(\frac{3}{\pi}\right)^{1/3}\left(\frac{1+X}{2m_{\rm H}}\right)^{4/3}.$$

We note that K_2 was formulated for electron degeneracy pressure, and the last factor was obtained by considering how many electrons are donated by the different elements in the star. For a gas of neutrons, this factor simply becomes unity, and the value of K_2 thus obtained can be combined with eqn. (20) from the week 4 lectures on polytropes to obtain an estimate of the maximum mass. The result of doing this gives $M = 5.26\ {\rm M}_\odot$. Prove this for yourselves!

Note that we have neglected a large amount of complex physics in making this estimate. First, we have neglected the interactions between the neutrons. Second, and this is very important, the gravitational field in a neutron star is so strong (i.e. the escape velocity from the surface of the star is starting to approach c, the speed of light) that we need to use Einstein's theory of gravity and not Newton's to get an accurate estimate of the maximum mass of a neutron star, since Einstein's theory has the effect of making the gravitational field stronger. The result of including these effects is to reduce the maximum mass of a neutron star down to $\approx 3\ {\rm M}_\odot$. Once a neutron star exceeds this mass, it appears that it must inevitably collapse to form a black hole.

1.2.3 Observations of neutron stars

Observationally, neutron stars have been detected in the form of the *pulsars*, which emit pulses at very regular intervals, with periods between a few milliseconds and a few seconds. These are most often observed in radio emission. The interpretation of the observations is that they originate from a rotating neutron star, which is predominantly radiating in specific directions; a pulse is observed when the beam of radiation sweeps past the observer.

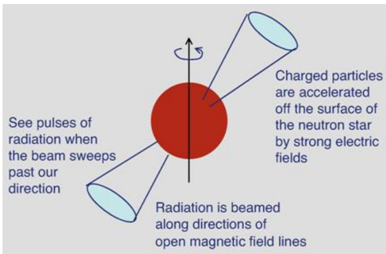

Figure 2: