3C74: TOPICS IN MODERN COSMOLOGY

Problem Sheet 4: Answers to be handed in by 22 March 2007

Question 1

(i) Write down the reaction that converts neutrons to protons in the early Universe when the particles are in thermal equilibrium.

(ii) Explain briefly why thermal equilibrium will come to an end.

(iii) In the cosmic production of ^{4}He, the neutron half-life effectively determines the amount of ^{4}He produced. If the neutron half-life was 900 s instead of the usual value of 614 s, calculate (using the relevant equations in Liddle's book, Chapter 12) the resulting mass-fraction Y_4 of He.

Question 2

(i) The Friedmann equation can be written as an equation showing how the density parameter Ω varies with time

$$|\,\Omega(t) - 1\,| \;=\; \frac{|\,k\,|}{a^2\,H^2}.$$

Using the solutions for radiation- and matter-dominated Universes ($a \propto t^{1/2}$; $a \propto t^{2/3}$), show how Ω varies with time, and explain the ensuing "flatness problem".

(ii) Assuming a radiation-dominated Universe, estimate how close Ω was to unity at an age of $t = 10\,\mathrm{s}$, if today ($t_0 = 1.4 \times 10^{10}\,\mathrm{yr}$) we measure $\Omega_0 = 0.3$ and if we measure $\Omega_0 = 0.9$.

(iii) By considering the Friedmann equation as given above, explain how the concept of inflation can solve the flatness problem.

Question 3

(i) The density contrast of a fluctuation $\delta(t)$ evolves according to the expression:

$$\ddot{\delta} + \frac{2\,\dot{a}\,\dot{\delta}}{a} - 4\pi G\rho\delta = 0$$

where $a(t)$ is the scale factor, ρ is the mean density, and G is the gravitational constant. Discuss briefly the role of the second term, and qualitatively the growth of perturbations in a static universe.

(ii) Show that a solution in an Einstein-de Sitter ($\Omega = 1$) universe, at which $a \propto t^{2/3}$,is:

$$\delta = A\,t^{\frac{2}{3}} \, + \, B\,t^{-1}$$

and discuss the physical significance of the two terms. Explain why such fluctuations stop growing in a low density Universe at a certain redshift.

Question 4

Assume a power-spectrum of density fluctuations of the form

$$\langle|\delta_{\mathbf{k}}|^2\rangle \propto k^n\ ,$$

where k is a wave number and n is a constant. By dimensional arguments derive the rms density and potential fluctuations in a sphere of radius R.