3C74: TOPICS IN MODERN COSMOLOGY

Problem Sheet 3: Answers to be handed in by 1 March 2007

[1] Show that in a spatially flat Universe, $\Omega_k = 0$, $\Omega_m + \Omega_\Lambda = 1$, the age of the Universe can be written as

$$t(z) = \frac{2}{3H_0\Omega_\Lambda^{1/2}} \ln\left[\frac{1+\cos\theta}{\sin\theta}\right] ,$$

where

$$\tan\theta = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/2} (1+z)^{3/2} .$$

Hence show that the present age of the Universe is given by

$$t_0 = \frac{2}{3H_0\Omega_\Lambda^{1/2}} \ln\left[\frac{1+\Omega_\Lambda^{1/2}}{(1-\Omega_\Lambda)^{1/2}}\right] .$$

[2] Derive and sketch the angular diameter vs. z relation (out to $z \gg 1$) for "rigid rods" in Friedmann model with $\Omega_m = 1$. At what z does the minimum angular diameter occur in this $\Omega_m = 1$ case? What is the physical interpretation?

[3] Radiation in the present universe, corresponding to a thermal background at 2.725 K, contributes an energy density corresponding to a density parameter of $\Omega_{\rm rad} = 2.47 \times 10^{-5} h^{-2}$. The typical energy of a photon in a thermal distribution is given by $3k_BT$ (where $k_B = 8.6 \times 10^{-5}$ eV K^{-1}). Assuming that the number of neutrinos equals the number of photons, calculate the mass-energy (in eV) that these neutrinos would need to contribute a critical density. [Assume that the thermal energy of the neutrinos is negligible compared to their mass-energy.]

Briefly discuss whether the neutrino can be considered a serious dark matter candidate by comparing your derived mass-energy with recent experimental values for the neutrino mass.