3C74: TOPICS IN MODERN COSMOLOGY

Problem Sheet 2: Answers to be handed in by 15 February 2007

Question 1

(a) Explain what is meant by the deceleration parameter q_0.
(b) By considering a Taylor expansion of the scale factor a about the present time t_0, derive an expression for q_0.
(c) By considering the full acceleration equation as given by

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3}$$

show that for a pressure-less universe, the deceleration parameter q_0 is given by

$$q_0 = \frac{\Omega_0}{2} - \Omega_\Lambda(t_0)$$

(d) Show that a radiation-dominated universe with $\Lambda = 0$ has a deceleration parameter $q_0 = \Omega_0$.

Question 2

(a) Estimate the age of the Milky Way by considering the radioactive decay of Uranium using the following data. Uranium is produced in supernovae and the initial abundances of the two isotopes U^{235} and U^{238} are expected to be in the ratio $\mathrm{U}^{235}/\mathrm{U}^{238} \simeq 1.65$ while the present abundance ratio is $\simeq 0.0072$. The decay rates of the isotopes are $\lambda(\mathrm{U}^{235}) = 0.97 \times 10^{-9}\,\mathrm{yr}^{-1}$ and $\lambda(\mathrm{U}^{238}) = 0.15 \times 10^{-9}\,\mathrm{yr}^{-1}$ and the decay law is $U(t) = U(0)\,\exp(-\lambda t)$.

(b) By assuming that the Universe is flat with present-epoch matter density paremeter $\Omega_m = 1$, and that the Milky Way took a minimum of an additional $10^9\,\mathrm{yr}$ to form in the first place, obtain an upper limit on the value of the Hubble constant H_0. How would your result change if the universe is flat with $\Omega_m = 0.3$?