3C74: TOPICS IN MODERN COSMOLOGY

Problem Sheet 1: Answers to be handed in by 1 February 2007

Question 1

Using the relation between apparent magnitude m, absolute magnitude M and luminosity distance d_L:

$$m - M = 5\log_{10}\left(\frac{d_L}{10\text{ pc}}\right),$$

estimate the luminosity distance d_L to which the following objects could be seen if $m = 20$: (i) a star like the sun ($M = 4.72$); (ii) a globular cluster; (iii) a bright galaxy.

Question 2

The Friedmann equation describing the expansion of the Universe (with $\Lambda = 0$) is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

and the fluid equation, describing the evolution of the density ρ of material in the Universe is given by

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0.$$

Using these two equations, derive the acceleration equation which describes the acceleration of the scale factor a and is given by

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right).$$

Question 3

Using the Friedmann equation as given above, determine the present density ρ_0 (in SI units) of the Universe asuming that it is flat and the Hubble constant $H_0 = 70\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$.

Estimate the present observed density of the Universe (in SI units) by supposing that the Milky Way is a typical galaxy containing 10^{11} stars (each weighing one solar mass) and that galaxies are typically separated by a distance of one megaparsec.

Compare the two densities you have derived and briefly comment on what this indicates about the type of Universe we live in.

[$1\,\text{M}_\odot = 1.989 \times 10^{30}\,\text{kg}$; $1\,\text{pc} = 3.086 \times 10^{16}\,\text{m}$; $\text{G} = 6.672 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$]