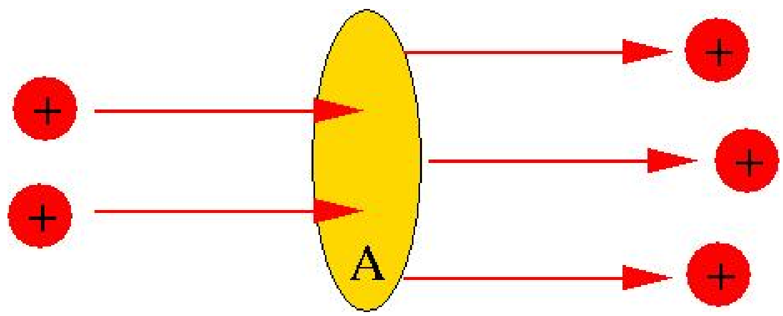
5 DC circuits

5.1 Electric current

- The current through a surface A is defined as the net charge that passes through the surface per second.

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} \tag{5.1}$$



where dQ is the amount of positive charge that passes through A.

- Recall that in electrostatic equilibrium there are no electric fields or currents inside a conductor, so no moving charges. All charges are arranged on the surface. For currents to exist in the volume of a conductor, there must be a source of electric field that drives charges into motion and maintains that motion.

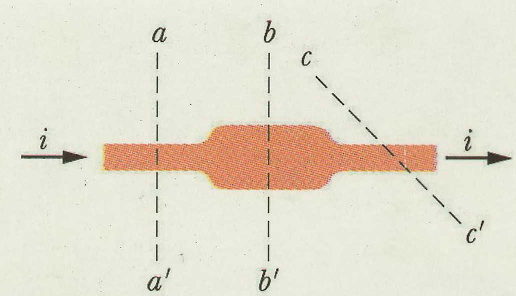


Fig. 47 The current i through the conductor has the same value at planes aa', bb', and cc'.

- Question: in the conductor shown in Figure 47, what current passes through the planes aa′, bb′, cc′?
- Answer: the same current, because of charge conservation.

- The SI unit of current is the ampere (A): one ampere = one coulomb per second ($1 \text{ A} = 1 \text{ Cs}^{-1}$).

5.2 A microscopic model of current

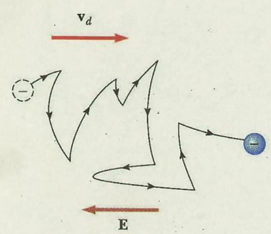


Figure 48 A schematic representation of the zigzag motion of an electron in a conductor. The changes in direction are the result of collisions between the electron and atoms in the conductor. Note that the net motion of the electron is opposite the direction of the electric field. Each section of the zigzag path is a parabolic segment.

- Current flows if a potential difference (and hence and electric field $\underline{E}$) exists across a conductor. An electron responds by moving with a net drift velocity, $\underline{v}_{\mathrm{d}}$ in the opposite direction to $\underline{E}$, as in Figure 48.
- Although it is electrons that move, conventionally we imagine equivalent positive charge carriers $+q$ moving with a velocity $\underline{v}_{\mathrm{d}}$ parallel to $\underline{E}$.
- We define the **current density** as a vector in the direction of $\underline{v}_{\mathrm{d}}$ whose magnitude is the net amount of charge crossing a unit area perpendicular to the drift velocity in unit time, so

$$\boxed{\underline{J} = nq\underline{v}_{\mathrm{d}} \qquad (5.2)}$$

with n the number of charge carriers per unit volume.

The current flowing through an area A is then

$$\boxed{I = \int_A \underline{J} \cdot \mathrm{d}\underline{a} \qquad (5.3)}$$

5.3 Ohm's law

Georg Ohm (1787-1854) found experimentally that for many materials, including most metals, the current density is proportional to the applied electric field,

$$\underline{J} = \sigma \underline{E} \tag{5.4}$$

where σ is a constant called the conductivity of the conductor. Equation 5.4 is called **Ohm's law**. The conductivity σ, as well as its inverse, the resistivity ρ, defined by

$$\rho = \frac{1}{\sigma} \tag{5.5}$$

does not depend on the geometry of the conductor but is a property of the material from which the conductor is made. Resistivity has units of ohm metres (Ω m). Some typical values are:

Material	Resistivity, $\rho(\Omega m)$
Copper	1.7×10^{-8}
Aluminium	2.82×10^{-8}
Iron	10×10^{-8}
Carbon	3.5×10^{-5}
(Glass)	10^{10} to 10^{14}

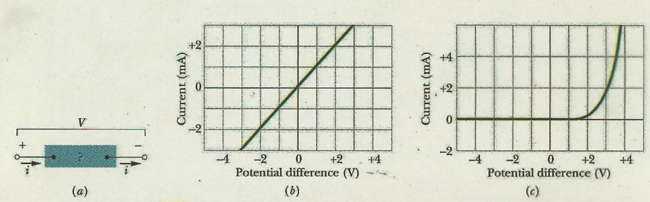


Fig. 51 (a) A potential difference V is applied to the terminals of a device, establishing a current i. (b) A plot of current i versus applied potential difference V when the device is a 1000 Ω resistor. (c) A plot when the device is a semiconducting pn junction diode.

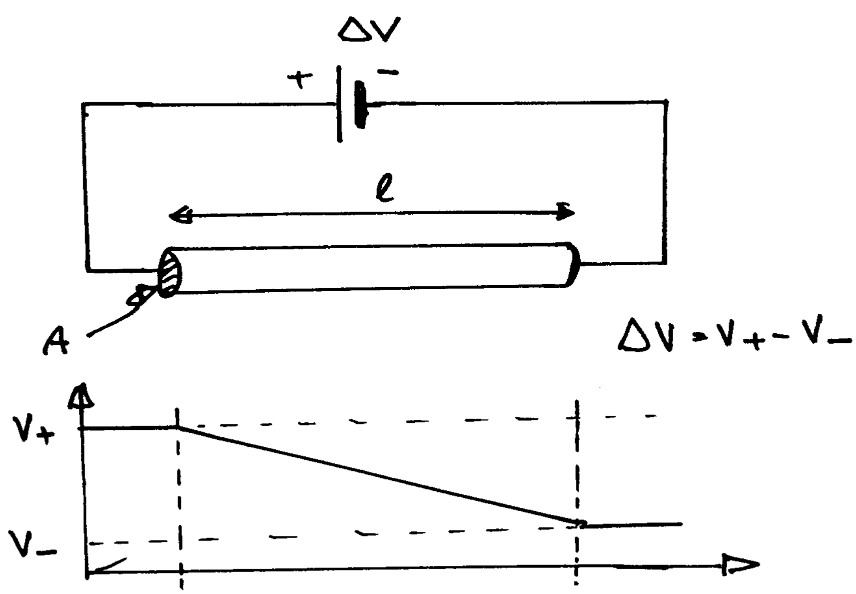
The final entry shows why glass is used as an electrical insulator on telephone poles etc.

- Ohmic materials are materials that obey Ohm's law - they include the resistors commonly used in electrical circuits.
- Non-ohmic materials include semiconductors (see Figure 51), frequently used in digital logic devices, such as pocket calculators.

Consider now the case of a wire of length l and cross-sectional area A. A uniform field $\underline{E}$ is associated with a constant potential difference ΔV:

$$E = \frac{\Delta V}{l} \tag{5.6}$$

(this comes from $\underline{E} = -\nabla V$ in one dimension, ie $E_x = -\frac{\partial V}{\partial x}$).



- Thus we have, combining 5.3, 5.4 and 5.6

$$J = \frac{I}{A} = \frac{\sigma \Delta V}{l} \tag{5.7}$$

$$\boxed{\Delta V = I \left(\frac{l}{\sigma A}\right) = I\ R \qquad (5.8)}$$

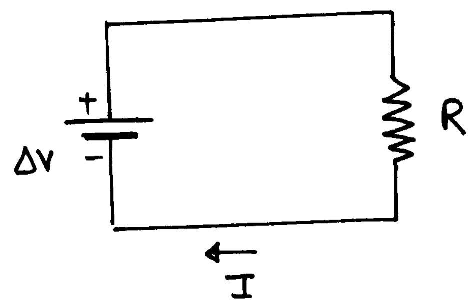
where

$$R = \frac{l}{\sigma A} \tag{5.9}$$

is the resistance of the conductor. Its SI unit is the ohm (Ω). One ohm = one volt per ampere ($1\Omega = 1$ V A^{-1}).

5.4 Electrical energy and power

- Consider the following circuit:



A battery connected by wires (of negligible resistance) to a resistor of resistance R, maintaining a potential difference ΔV across R. From Ohm's law a steady current I flows.

- The current I corresponds to the charge flowing per unit time, so in a time dt, a charge d$Q = I$dt flows through he resistor R.

- This charge changes potential by ΔV as it moves through the resistor and therefore loses electric potential energy

$$\mathrm{d}U = \mathrm{d}Q\Delta V = I\mathrm{d}t\Delta V \tag{5.10}$$

- We define power as the rate of change of potential energy,

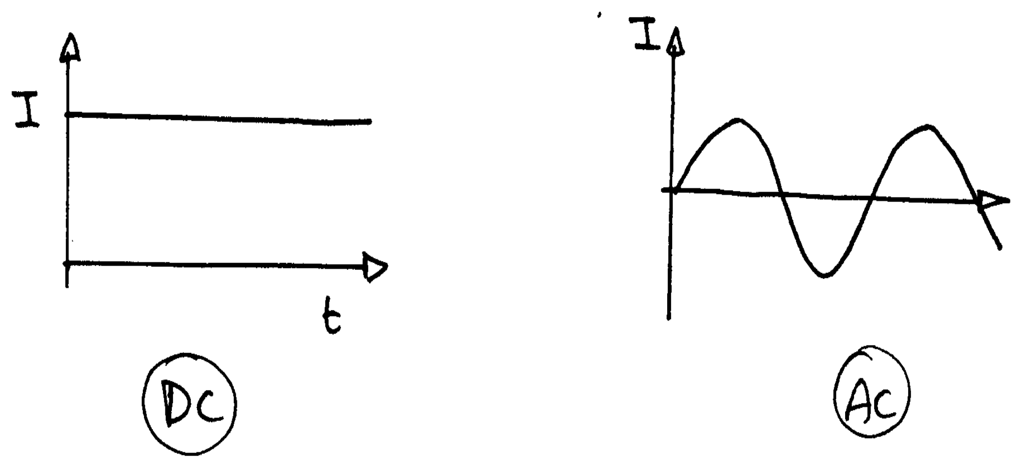
$$P = \frac{\mathrm{d}U}{\mathrm{d}t} = I\Delta V \tag{5.11}$$

- In the circuit, chemical energy from the battery is continually being converted to internal thermal energy in the resistor. The course of an electron through R is like that of a stone falling through water at constant terminal speed. Its average kinetic energy is constant (current) but it loses potential energy to its surroundings. In the case of the electrons making up the current, the energy is lost by collisions with the (static) ions in the metal, generating heat.

- The unit of power is the <u>watt</u> (W): one watt = one volt-ampere = one joule per second (1 W = 1 V A = 1 J s^{-1}).

5.5 AC and DC current

- 'DC' refers to 'direct current', as opposed to 'alternating current' (AC).

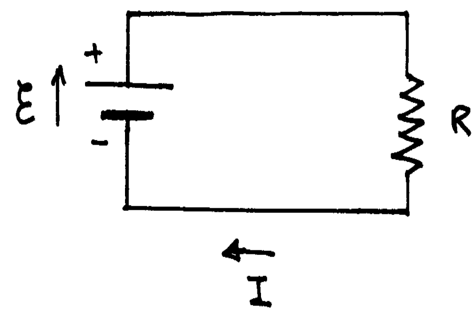


5.6 Pumping charges: EMF

- To maintain a constant current in a circuit we need a 'charge pump' to keep the potential difference between a pair of terminals. Examples of such pumps include batteries and electrical generators such as dynamos and solar cells.
- Consider a simple battery and resistor circuit:
- Charges (positive) flow clockwise, from the positive to negative battery terminals, through R. To <u>pump</u> charge (dq say) from the negative to positive terminals (ie <u>inside</u> the battery) requires <u>work</u> (dW say).
- We define the 'EMF' (symbol $\mathcal{E}$) of the battery to be the work done per charge:

$$\mathcal{E} = \frac{\mathrm{d}W}{\mathrm{d}q} \tag{5.12}$$

  where EMF stands for 'electromotive force' the 'force' that makes the charges move.
- Equation 5.12 is essentially the same as the definition of potential difference ΔV. Note that $\mathcal{E}$ is therefore <u>not</u> a force but a potential difference.



5.7 Single-loop circuits

- We can apply the principle of conservation of energy to allow us to calculate currents in a circuit.
- In section 5.4 we saw that power

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = I \; \Delta V \tag{5.13}$$

- In the circuit above $\Delta V = I \; R$, so

$$P = I^2 \; R \tag{5.14}$$

is the rate of energy dissipation in the resistor. Substituting in equation 5.13 we have for the energy dissipated in a time dt,

$$\begin{aligned} \mathrm{d}W &= I^2 \; R\mathrm{d}t && (5.15) \\ &= \mathcal{E}\mathrm{d}q && (5.16) \\ &= \mathcal{E}I\mathrm{d}t && (5.17) \end{aligned}$$

$$\therefore \mathcal{E}I = I^2R \tag{5.18}$$

$$\boxed{\mathcal{E} = IR \qquad (5.19)}$$

- The (chemical) energy per charge gained in the battery equals the (thermal) energy per charge dissipated in the resistor.

- Another way of expressing this is to say that "the sum of all potential differences around any closed circuit loop must be zero"

$$\boxed{\sum_{\text{closedloop}} \Delta V = 0 \qquad (5.20)}$$

which is known as Kirchhoff's loop rule.

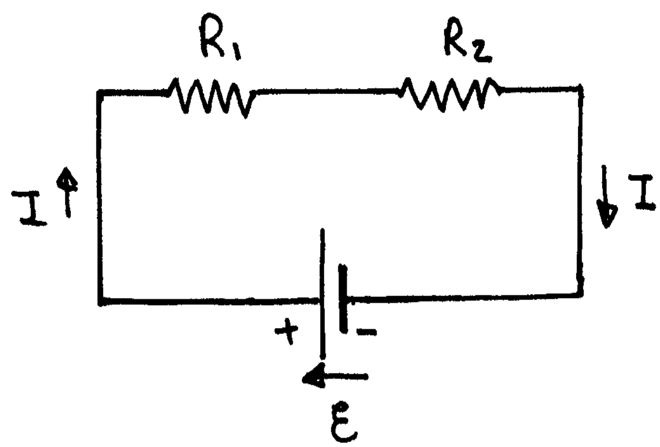
- In the simple circuit composed of a resistor and a battery, this is simply

$$\mathcal{E} - IR = 0 \tag{5.21}$$

$$\therefore \mathcal{E} = IR \tag{5.22}$$

as in equation 5.19

- For resistors in series



Kirchhoff's loop rule gives

$$\mathcal{E} - IR_1 - IR_2 = 0 \tag{5.23}$$

which is like equation 5.19 with $R_{\text{equivalent}} = R_1 + R_2$. In general for resistors wired in series

$$R_{\text{equivalent}} = \sum_i R_i \tag{5.24}$$

- The change in electric potential around such a circuit is illustrated in Figure 52.

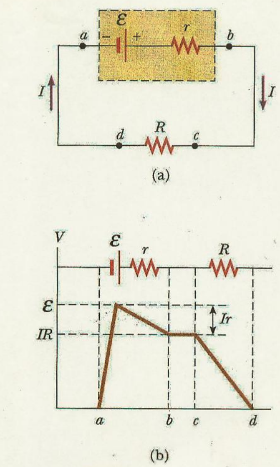


Figure 52 (a) Circuit diagram of a source of emf $\mathcal{E}$ (in this case, a battery), of internal resistance r, connected to an external resistor of resistance R. (b) Graphical representation showing how the electric potential changes as the circuit in part (a) is traversed clockwise.

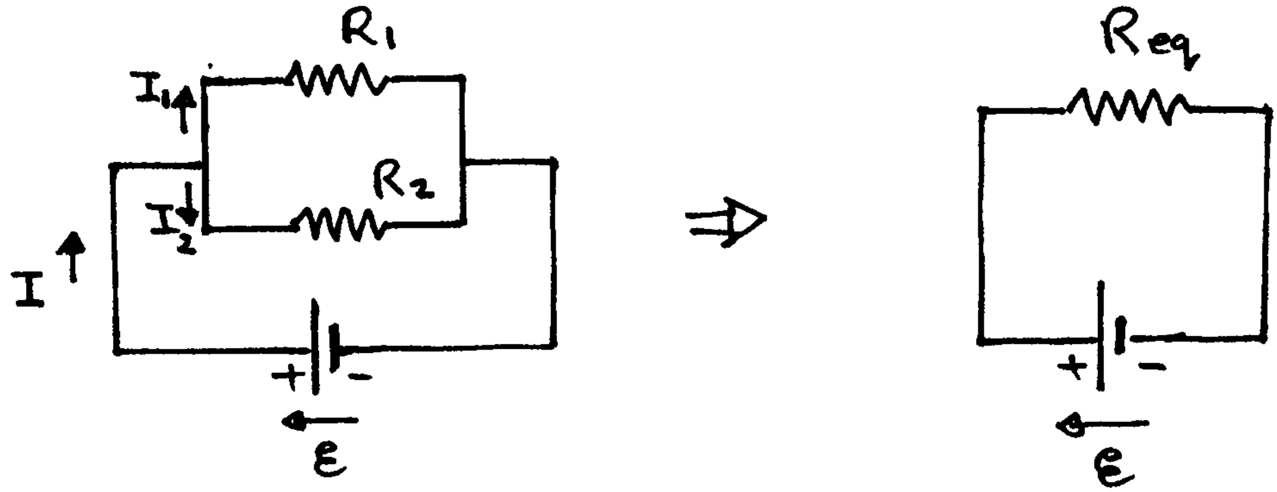
5.8 Multi-loop circuits

- What is the equivalent resistance for a circuit with resistors in parallel?
- We note first that the potential differences across R_1 and R_2 are the same.
- We next note that "The sum of the currents entering any junction in a circuit must equal the sum of currents leaving it"

$$\sum I_{\text{in}} = \sum I_{\text{out}} \tag{5.25}$$

which is Kirchhoff's junction rule. It follows from the conservation of charge in the circuit.

- So in the circuit considered above, and consisting of two resistors in series and a



battery, we have:

$$
\begin{aligned}
I &= I_1 + I_2 && (5.26)\\
&= \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} && (5.27)\\
&= \Delta V \left(\frac{1}{R_1} + \frac{1}{R_2} \right) && (5.28)\\
&= \Delta V \frac{1}{R_{\text{eq}}} && (5.29)\\
\therefore \frac{1}{R_{\text{eq}}} &= \frac{1}{R_1} + \frac{1}{R_2} && (5.30)
\end{aligned}
$$

or, in general,

$$
\boxed{\frac{1}{R_{\text{equivalent}}} = \sum_i \frac{1}{R_i} \qquad (5.31)}
$$

for a collection of resistors wired in parallel.

5.9 Time-varying currents

Figure 53 shows an 'RC series circuit', a resistor and a capacitor in series.

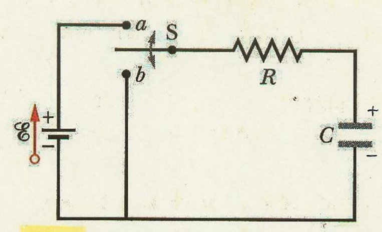


Fig. 53 When switch S is closed on *a*, the capacitor is *charged* through the resistor. When the switch is afterward closed on *b*, the capacitor *discharges* through the resistor.

- To charge the capacitor C from zero, we close the switch S to point a. From Kirchhoff's loop rule

$$\mathcal{E} - IR - \frac{q}{C} = 0 \tag{5.32}$$

where $I = I(t)$ and $q = q(t)$ both vary with time and across the capacitor $\Delta V = \frac{q}{C}$. Note that the potential decreases on going from the positive to the negative plate of the capacitor. We also have $I = \frac{\mathrm{d}q}{\mathrm{d}t}$, so equation 5.32 becomes

$$\frac{\mathrm{d}q}{\mathrm{d}t} + \left(\frac{1}{RC}\right) q = \frac{\mathcal{E}}{R} \tag{5.33}$$

which has the solution

$$q(t) = C\mathcal{E}\left(1 - \mathrm{e}^{\frac{-t}{RC}}\right) \tag{5.34}$$

from which we can derive the current

$$\boxed{I(t) = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{\mathcal{E}}{R}\mathrm{e}^{\frac{-t}{RC}} \qquad (5.35)}$$

where RC is the <u>time constant</u> of the circuit.

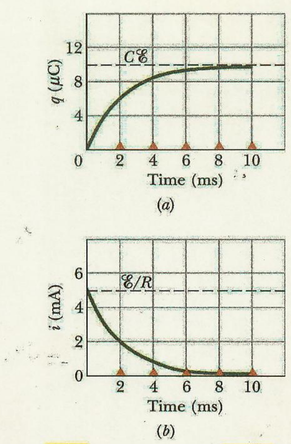


Fig. 54 (*a*) A plot of Eq. (17.13) which shows the buildup of charge on the capacitor of Fig. 53. (*b*) A plot of Eq. (17.14), which shows the decline of the charging current in the circuit of Fig. 53.. The curves are plotted for $R = 2000\ \Omega$, $C = 1\ \mu$F, and $\mathscr{E} = 10$ V: the small triangles represent successive intervals of one time constant τ.

- Plots of $q(t)$ and $I(t)$ are shown in Figure 54.
- Note that at the time $t = RC$,

$$\frac{I(t = RC)}{I(t = 0)} = \frac{\frac{\mathcal{E}}{R}\mathrm{e}^{-1}}{\frac{\mathcal{E}}{R}} = \frac{1}{\mathrm{e}} = 0.368 \tag{5.36}$$

- The units of RC are Ω F = V A^{-1} C V^{-1} = V C^{-1} s C V^{-1} = s