4 Dielectrics

We have seen that when a conductor is placed in an external field, conduction electrons move until induced charges ensure that the macroscopic electric field is zero everywhere inside the conductor.

In insulating materials, or **dielectrics** as they are often called, all electrons are bound to particular atoms, but applied electric fields still slightly displace the electrons in each atom. This displacement results in the appearance of induced charges which reduce the field in the insulator, though not completely canceling it. In this chapter we introduce the basic properties of dielectrics and discuss the microscopic origin of these properties.

4.1 Polarization and dielectric constant

An ideal dielectric material has no free charges. However the material constituents, atoms and molecules, are affected by the presence of an electric field. To be specific, consider for example a dielectric like argon (the different types of dielectric will be discussed in the following). The electric field causes the displacement of the electrons and of the nucleus, in opposite direction. From a macroscopic point of view, this can be visualized as a displacement of the entire positive charge in the dielectric relative to the negative charge. The dielectric is said to be **polarized**.

To be more quantitative, consider a small element Δv of a dielectric medium that is, as a whole, electrically neutral. If the medium is polarized, than a separation of positive and negative charge has been effected, and the volume element is charaterized by an electric dipole moment $\Delta \underline{p}$. The **electric polarization**, or simply polarization, of the medium $\underline{P}$ is defined as the **electric dipole moment per unit volume**:

$$\underline{P} = \frac{\Delta \underline{p}}{\Delta v} \; . \tag{4.1}$$

The degree of polarization depends on the applied electric field $\underline{E}$, and on the properties of the constituents (atoms or molecules) of the dielectric material. We limit here our discussion to materials for which $\underline{P}$ vanishes when $\underline{E}$ vanishes. This is the behaviour of most dielectric materials. We also assume the material to be isotropic, which implies that the polarization has the same direction as the electric field which causes it. Under these assumptions, and in the limit of weak applied electric field we can write

$$\underline{P} = \chi \underline{E} \tag{4.2}$$

where χ is called the **electric susceptibility** of the material. A related quantity is the dimensionless **dielectric constant** κ (Greek letter kappa), defined as

$$\kappa = 1 + \frac{\chi}{\epsilon_0}\ . \tag{4.3}$$

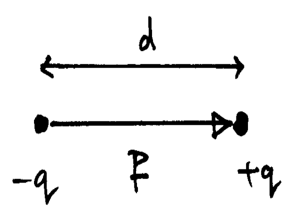
κ is sometimes also called 'relative permittivity' and given the symbol ϵ_{r}.

Some values for κ:

Material	κ	
Vacuum	1	exactly
Argon gas	1.00052	
Dry air	1.00059	often approximated as 1.0
Silicone oil	2.5	
Pyrex glass	5.6	
Porcelain	6	
Water	80	large!
Strontium titanate	233	large!

4.2 An electric dipole in an electric field

To better understand the origin of the polarization, first consider a dipole in a uniform field $\underline{E}$:

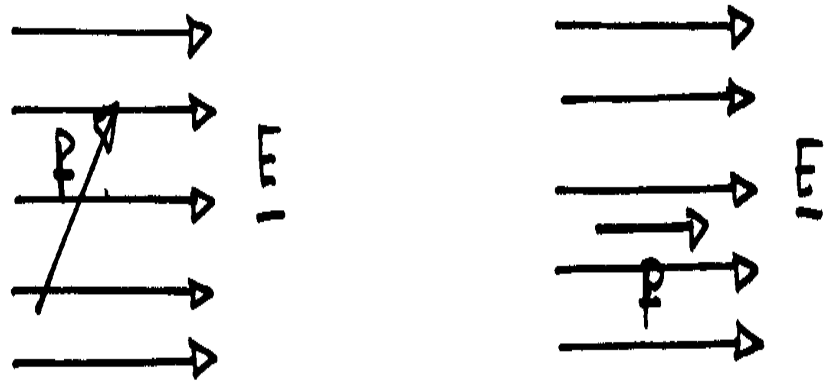


We define the electric dipole moment, $\underline{p}$ as a vector of magnitude qd directed from $-q$ to $+q$.

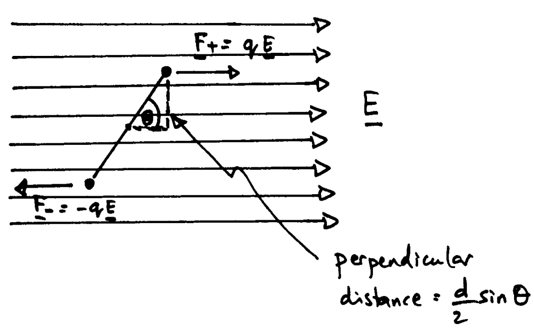
In a field the dipole experiences a torque aligning it so that $\underline{p}$ is parallel to $\underline{E}$.

The torque acting on the dipole is

$$\underline{\tau} = \sum_i \underline{r}_i \times \underline{F}_i$$



where the sum is extended to the two charges, and $\underline{r}_i$ is the position of the i-th charge from the center of the dipole.



$$\tau = \underline{r}_+ \times (+qE) + \underline{r}_- \times (-q\underline{E}) \tag{4.4}$$

$$\tau = q(\underline{r}_+ - \underline{r}_-) \times \underline{E} \tag{4.5}$$

which can be written

$$\boxed{\underline{\tau} = \underline{p} \times \underline{E} \qquad (4.6)}$$

The potential energy U of the dipole is a minimum when $\underline{p}$ is parallel to $\underline{E}$ and a maximum when $\underline{p}$ is perpendicular to $\underline{E}$. The work done to rotate the dipole from θ_i to θ_f equals the change in U:

$$U_f - U_i = \text{work done} = \int_{\theta_i}^{\theta_f} |\underline{\tau}| \mathrm{d}\theta = pE \int_{\theta_i}^{\theta_f} \sin\theta \mathrm{d}\theta \qquad (4.7)$$

$$U_f - U_i = -p\ E(\cos\theta_f - \cos\theta_i) \qquad (4.8)$$

By convention we choose $U_i = 0$ at $\theta_i = 90^{\mathrm{o}}$ so that

$$U_f = -p\ E\cos\theta_f \qquad (4.9)$$

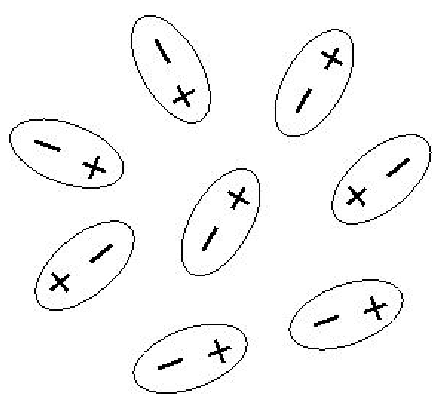
or in vector notation

$$\boxed{U = -\underline{p}\cdot\underline{E} \qquad (4.10)}$$

4.3 The origin of the polarization

The molecules of a dielectric may be classified as polar or non polar.

1. A **polar molecule** is one that has a permanent dipole moment, even in the absence of an applied electric field $\underline{E}$. A polar molecule consists at least of two different species of atoms. Examples: water and strontium titanate (an ionic crystal) have an intrinsic and large dipole moment $\underline{p}$. In the absence of an electric field, a macroscopic



piece of the polar dielectric is not polarized, since the individual dipoles are randomly oriented (see Figure). If the polar dielectrics is subjected to an electric field, the individual dipoles experience torques which tend to align them with the field. A macroscopic polarization is created.

2. **Nonpolar molecules** do not have permanent dipole moments, i.e. the "centers of gravity" of the positive and negative charge distribution normally coincide. Symmetrical molecules such as H_2, N_2, O_2 and monoatomic molecules such as He, Ne,

and Ar fall into this category.

The application of an electric field causes a relative displacement of the positive and negative charges, and the molecular dipoles so created are called **induced dipoles**.

4.4 Dielectric capacitors

In 1837 Faraday found that if he filled the space between the plates of a capacitor with oil, its capacitance increased. This property is general to all dielectrics, and it is possible to show that

$$\boxed{C_{\text{dielectric between plates}} = \kappa\, C_{\text{vacuum between plates}} \qquad (4.11)}$$

and the energy density of the field has increased

$$\boxed{U_{\text{dielectric}} = \frac{1}{2}\, \kappa \epsilon_0 E^2 \qquad (4.12)}$$

Capacitance is not the only quantity modified by κ, in fact the general rule is - "In a region filled by a dielectric material of dielectric constant κ, all the electrostatic equations containing the permittivity constant ϵ_0 are to be modified by replacing ϵ_0 with $\kappa\epsilon_0$".